Nonlinear derating of high-intensity focused ultrasound beams using Gaussian modal sums

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A method is introduced for using measurements made in water of the nonlinear acoustic pressure field produced by a high-intensity focused ultrasound transducer to compute the acoustic pressure and temperature rise in a tissue medium. The acoustic pressure harmonics generated by nonlinear propagation are represented as a sum of modes having a Gaussian functional dependence in the radial direction. While the method is derived in the context of Gaussian beams, final results are applicable to general transducer profiles. The focal acoustic pressure is obtained by solving an evolution equation in the axial variable. The nonlinear term in the evolution equation for tissue is modeled using modal amplitudes measured in water and suitably reduced using a combination of “source derating” (experiments in water performed at a lower source acoustic pressure than in tissue) and “endpoint derating” (amplitudes reduced at the target location). Numerical experiments showed that, with proper combinations of source derating and endpoint derating, direct simulations of acoustic pressure and temperature in tissue could be reproduced by derating within 5% error. Advantages of the derating approach presented include applicability over a wide range of gains, ease of computation (a single numerical quadrature is required), and readily obtained temperature estimates from the water measurements.

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I. INTRODUCTION

In the design and testing of high-intensity focused ultrasound (HIFU) devices, as well as in treatment planning of focused ultrasound surgery, it is important to know the acoustic pressure and temperature fields in the tissue of interest. Pressure measurements are much more readily performed in water than in in vivo models, ex vivo samples, and even tissue phantoms. Techniques for converting measurements in water to estimates appropriate for tissue, i.e., derating techniques, can be valuable for characterizing HIFU systems.

The simplest derating technique involves taking measured output acoustic pressure in water and reducing it by a factor based upon typical tissue attenuation, for example, 0.3 dB/cm/MHz (Center for Devices and Radiological Health, 2008). This is typically applied to the center frequency of pulses used in diagnostic ultrasound applications. Derating using a single multiplicative factor can be effective for low amplitude ultrasound beams, where generation of higher harmonics due to nonlinear propagation effects is not present. During the propagation of finite-amplitude sound beams, energy is transferred between the harmonic modes due to nonlinear self-interaction; energy is also scattered and converted into heat at rates that depend on the frequency of the modes. This interplay of energy transfer and loss makes derating a highly nontrivial task. It requires knowledge of the frequency dependence of material absorption, as studied by Verma et al. (2005a, 2005b) and Rielly et al. (2000).

In an attempt to account for broadband pulses or spectral broadening due to higher harmonic generation, Schafer (1990) introduced “wideband” derating, in which the waveform is Fourier transformed, multiplied by the appropriate frequency-dependent attenuation and corresponding phase velocity dispersion for tissue, and transformed back to the time domain. Schafer also considered other techniques, such as derating only the dominant spectral peak of the pulse, and derating the centroid of the pulse frequency spectrum. These methods were compared against data from propagation in water but not against tissue data. Szabo et al. (1999) observed that varying the power of pulses in water to match the peak negative acoustic pressure of pulses in a tissue mimicking material (TMM) resulted in close agreement of time average intensity and peak positive acoustic pressure. The results were better than the “wideband” derating approach of Schafer, but only useful with a priori knowledge of the TMM.

Taking advantage of the strong focusing typical of therapeutic beams, Bessanova et al. (2010) scaled the transducer acoustic power, rather than the attenuation, to derate acoustic pressure data in water. This was done to match the focal acoustic pressures in water and tissue, relying on the assumption that linear acoustic pressure gain $G$ is high enough that most of the nonlinear interaction occurs in the focal region. The derated acoustic pressure is computed...
using the scaled initial source amplitude. Agreement between derated water simulations and tissue simulations was reported as 35% for weakly focused diagnostic transducers ($G = 5–10$) and 5% for strongly focused ($G = 40–60$) therapeutic transducers.

An alternative method for derating measurements involves first decomposing the measurements into Gaussian modal sums (Wu and Du, 1990; Wu and Nyborg, 1992; Soneson and Myers, 2007). Soneson and Myers (2007) showed that the acoustic pressure field in some cases can be accurately modeled using an approximate solution to the Khokhlov–Zabolotskaya–Kuznetsov (KZK) equation in the form of a sum of Gaussian modes. The technique is well suited to the low-gain case. Additionally, Myers and Soneson (2009) derived a method for using the derated modal amplitudes to predict the temperature rise in the tissue of interest. While the results are derived in the context of a Gaussian weighted transducer, no assumption of Gaussian shading is required in the final results.

The next section demonstrates how derating is performed within the Gaussian-mode theory. Subsequently, simulated acoustic pressure measurements in water are used to obtain the modal amplitudes, which are then derated and used to construct estimates of acoustic pressure and temperature in tissue. The results are compared with direct simulation of the acoustic pressure and temperature fields in the tissue, using the full KZK equation.

### II. NONLINEAR DERATING MODEL

We consider an acoustic pressure field periodic in time, written as

$$p(r, z, t) = \frac{1}{2} \sum_{n=1}^{\infty} \left[ A_n(r, z) e^{i\omega_n t} + A_n^*(r, z) e^{-i\omega_n t} \right]. \quad (1)$$

Here $p$ is the deviation from ambient pressure, $z$ and $r$ are the axial and radial coordinates, $\omega_n$ is the angular frequency, and $t$ is time measured in the reference frame propagating at the small-signal sound speed $c_0$. In terms of the scaled variables $\zeta = z/d$, $\rho = r/a$, and $u_0 = A_0 / p_0$, where $d$ is the transducer focal length, $a$ the effective radius, and $p_0$ the peak acoustic pressure at the transducer face, the KZK equation (Hamilton and Morley, 1998) governing propagation of the HIFU beam may be written as (Soneson and Myers, 2007)

$$\frac{\partial u_n}{\partial \zeta} + \frac{i}{4nG} \nabla^2 u_n + \gamma_n u_n = \frac{inN}{4} \sum_{m=1}^{\infty} u_m(u_{m,n} + 2u_{m,n-1}), \quad n = 1, 2, \ldots. \quad (2)$$

Here $G = \pi a^2 f / (\rho c_0)$ is the linear acoustic pressure gain (with $f = \omega / (2\pi)$), and $N = 2\pi \rho_0 f d f (\rho_0 c_0)^3$ is the coefficient of nonlinearity, where $\beta$ is the nonlinear parameter for the propagation medium and $\rho_0$ the mass density. The complex parameter $\gamma_n$ is defined by

$$\gamma_n = d x(f) - \frac{2id}{\pi} a(f) \log \left( \frac{f}{f_r} \right), \quad \eta = 1, \quad (3a)$$

where $\eta$ is the exponent in the frequency-dependent absorption:

$$x(f) = x_s(f) f^n. \quad (3c)$$

Here $x_s$ is the absorption at some reference frequency $f$, indicated by the asterisk subscript, often 1 MHz. In the computations, we will assume $\eta = 1$ for soft tissue and $\eta = 2$ for water.

Soneson and Myers (2007) showed that it is advantageous computationally to represent the acoustic pressure field by a sum of Gaussian modes:

$$u_n(\rho, \zeta) = a_n(\zeta) \exp \left[ -G b(\zeta) \rho^2 \right], \quad (4a)$$

with

$$b(\zeta) = \frac{1 - iG}{G - (G + i) \zeta}. \quad (4b)$$

In this case the KZK equation simplifies to the following ordinary differential equation:

$$\frac{d a_n}{d \zeta} + \left[ \gamma_n - \frac{G + i}{G - (G + i) \zeta} \right] a_n = \frac{inN}{4} \sum_{m=1}^{\infty} a_m(a_{m,n} + 2a_{m,n-1}). \quad (5)$$

Equation (5) describes the evolution of the acoustic pressure modes with axial distance. The terms in the square brackets represent the effects of absorption, phase velocity dispersion, and diffraction, while the right-hand side of the equation arises due to nonlinear propagation effects. Computing the field in this Gaussian-mode approximation is much faster than other radial discretizations. If the radial coordinate is discretized over $J$ nodes, then solutions using the Gaussian-mode method are obtained in $1/J$ the amount of computational time and memory required by finite difference, finite element, or spectral methods.

A Green-function solution to the bioheat equation (Nyborg and Wu, 1994; Nyborg, 1988; Myers and Soneson, 2009) can be used to derive an expression for the axial temperature arising from absorption of the ultrasound field in a medium possessing the absorption characteristics given in Eqs. (3b) and (3c). The equation for the focal temperature, in a form applicable to both Gaussian and non-Gaussian transducers, is (Myers and Soneson, 2009)

$$T(0, 1) = \frac{\alpha_0}{4K \rho_0 c_0} \sum_{n=1}^{\infty} n^\eta a_n^2(0, 1) r_{n, eff}^2 \log \left( 1 + \frac{4Kt}{r_{n, eff}^2} \right), \quad (6a)$$

where $K$ is the thermal conductivity and $r_{n, eff}$ is the effective radius of the nth mode. For a Gaussian transducer, $r_{n, eff}$ is the radius at which the beam intensity of the nth mode falls to $1/e$ of its on-axis value. For a non-Gaussian transducer, the main lobe of the intensity mode may be fitted to a

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Gaussian profile by determining the intensity at an off-axis location $r_s$, and computing $r_{n,\text{eff}}$ from (Myers and Soneson, 2009)

$$r_{n,\text{eff}} = r_s/\sqrt{\ln(I_n(0, \zeta)/I_n(r_s, \zeta))^{1/2}}. \quad (6b)$$

Pressure for non-Gaussian transducers may be computed from Eqs. (1) and (4), and the relationship $u_k = A_k/p_0$. The result for the focal acoustic pressure is re-assembled here:

$$p(\rho = 0, \zeta = 1, t) = \frac{p_0}{2} \sum_{n=1}^{\infty} \left[ a_n(1)e^{i\omega_{an}} + a_n^*(1)e^{-i\omega_{an}} \right]. \quad (7)$$

If the acoustic pressure-mode amplitudes $a_n(1)$ and the beam widths $r_{n,\text{eff}}$ appropriate for tissue can be estimated from values measured in water, Eqs. (6) and (7) can be used to compute the focal acoustic pressure and temperature in a tissue medium. The derating strategy for performing the conversion from water to tissue is as follows.

The right side of Eq. (5) represents the effects of nonlinear acoustic propagation. If nonlinear propagation effects in tissue can be estimated from the corresponding nonlinear effects in water, i.e., if the terms on the right side of Eq. (5) can be computed from measurements of the modal amplitudes in water, then Eq. (5) can be solved to obtain the tissue modes. To this end, we set

$$q_n(\zeta) = \frac{inN}{4} \sum_{m=1}^{\infty} w_m(\zeta)[w_{n-m}(\zeta) + 2w_{m-n}(\zeta)], \quad (8)$$

where $w_n$ is an estimate for $a_n$ obtained by making measurements in water in conjunction with a derating procedure. The function $q_n(\zeta)$ is assumed to be a known quantity. Computation of $q_n(\zeta)$ is discussed subsequently. Here we proceed with solution of the equation

$$\frac{da_n}{d\zeta} + h_n(\zeta)a_n = q_n(\zeta), \quad (9a)$$

where

$$h_n(\zeta) = \left[ \frac{\gamma_n - G + i}{G - (G + i)\zeta} \right]. \quad (9b)$$

To obtain a particular solution to (9a), we multiply both sides by $\exp(-\int_0^\zeta h_n(u)du)$, use (9b) in the integrand, and perform the integration. A homogeneous solution to (9a) can be readily obtained after moving $h_n(\zeta)a_n$ to the right side and dividing through by $a_n$. The resulting total solution is

$$a_n(\zeta) = a_n(0)e^{-\gamma_n\zeta} \frac{G}{G - (G + i)\zeta} + \int_0^\zeta e^{-\gamma_n(\zeta')} \frac{G - (G + i)\zeta'}{G - (G + i)\zeta} q_n(\zeta')d\zeta'. \quad (10)$$

Equation (10) provides the mechanism by which the modal amplitudes are obtained. The first term, equal to the total solution when $N = 0$, is the solution in the limit of linear acoustics. It predicts that the focal acoustic pressure amplitude is simply the amplitude at the transducer $a(0)$, scaled by the attenuating factor $\exp(-\alpha d)$, and multiplied by the gain. In the presence of nonlinear propagation effects ($N > 0$), the second term in Eq. (10) contributes the modal amplitudes for the higher harmonics.

The critical step in solving Eq. (10) is obtaining the approximate modal amplitudes $w_n$ contained in the source term $q_n(\zeta)$. The procedure for determining the $w_n$ values involves measurement of pressure amplitudes in water, e.g., by a hydrophone scan along the $z$ axis. Devising the protocol for performing the water measurements, and for incorporating the water measurements in a manner that yields the best approximation to pressure measurements in tissue, is a primary objective of the nonlinear derating algorithm.

One method for performing the nonlinear derating is to make water measurements using the same source pressure as in tissue, then to multiply the $n$th amplitude by $\exp(-n\alpha z)$, where $\alpha$ is the tissue absorption. This is similar to the wideband derating performed by Schafer (1990). This type of derating, which we label “endpoint scaling” since amplitude reduction is performed after the modes have evolved from the transducer to the axial endpoint, is a logical extension of linear derating. However, since the harmonics are not attenuated during their evolution to the extent they would be in tissue, nonlinear effects can be overestimated. To address this problem, endpoint scaling is a combined with “source scaling,” a version of which was introduced by Bessanova et al. (2010). With source scaling, the experiments in water are performed at a lower value of the nonlinearity parameter in water than in tissue, by reducing the source acoustic pressure. That is, $N_w/N_t < 1$. When source scaling is performed, endpoint scaling by the full factor $\exp(-nz\alpha)$ can result in excessive amplitude reduction. Therefore, we propose combining source scaling with endpoint scaling by the factor $\exp(-n\alpha z)$, where $\alpha \leq 1$. Numerical experiments were performed to determine the relative amounts of source scaling (represented by $N_w/N_t$) and endpoint scaling (represented by $\alpha$) that produced derated amplitudes $w_n$ giving the best agreement with numerical simulations in tissue. Agreement was measured in terms of the maximum and minimum of the acoustic pressure waveform, and the temperature rise at the focus.

III. COMPUTATIONS

A. Computational design

Simulated acoustic pressure fields in water were obtained using the software HIFU_Simulator (Soneson, 2008), which solves the axisymmetric KZK equation for continuous wave beams using a split-step time and frequency domain algorithm on a finite difference discretization. HIFU_Simulator was also used to generate the focal acoustic pressures and temperatures in tissue against which the derating algorithm was compared. Transducer gains $G$ of 20, 40, and 60 were considered. A representative tissue attenuation, 55 dB/m at 1 MHz, was used for all simulations. Values of the coefficient of nonlinearity in tissue, $N_t$, equal to 0.1, 0.2, and 0.3 were considered. However, results are
presented only for $N_t = 0.3$, which represented the largest amount of nonlinearity and the biggest challenge to the derating algorithm.

A wide range of $N_w/N_t$ and $\varepsilon$ values were used in the simulations. For each value of $N_w/N_t$ considered, HIFU_Simulator was executed in a water environment using a source acoustic pressure that yielded a value of $N_w$ equal to $N_r$ (0.1, 0.2, or 0.3) times the current $N_w/N_r$. After the simulation completed, the acoustic pressure amplitudes at the focus (i.e., $z = d$ or $\zeta = 1$) were subsequently derated by the factor $\exp(-\varepsilon n \lambda d)$, yielding the derated water amplitudes $w_n$. The $w_n$ values were then used in (8) and (10) to compute the estimates of the tissue amplitudes, $a_n$. Numerical quadrature in (10) was performed using the trapezoidal rule. The modal intensities in the focal plane for water were plotted and the modal beam widths computed from (6b). A value of 0.4 mm was used for $r_s$. Once $a_n$ and $r_s$ were determined, the focal acoustic pressure and temperature in tissue could be constructed from (6) and (7).

The tissue acoustic pressures and temperatures determined through this derating process were compared with direct simulations in tissue performed with HIFU_Simulator. For acoustic pressure, an error metric was composed from the difference between direct simulation and derating in the peak positive acoustic pressures, added to the similar error in peak negative acoustic pressure. The error was normalized by the peak-to-peak acoustic pressure computed via direct simulation in tissue. That is,

$$\text{pressure error} = \frac{|P_{+\text{direct}} - P_{+\text{derating}}| + |P_{-\text{direct}} - P_{-\text{derating}}|}{P_{+\text{direct}} - P_{-\text{direct}}}.$$  \hspace{1cm} (11a)

The temperature error was the difference between the end of sonication temperature rise for direct simulation minus the end-of-sonication temperature for derating, normalized by the direct-simulation temperature rise:

$$\text{temperature error} = \frac{\Delta T_{\text{direct}} - \Delta T_{\text{derating}}}{\Delta T_{\text{direct}}}.$$  \hspace{1cm} (11b)

The sonication period was that required to raise the temperature about 60°F (to avoid boiling in a phantom), typically a few seconds, depending upon the value of the transducer gain.

B. Derating from reduced data

The function $q_n(\zeta)$ [Eq. (8)] capturing the effects of nonlinear propagation is typically small outside of the focal region. As a result, the integrand in (10) can be negligibly small away from the focus. Hence, it is often not necessary to determine the water modal amplitudes along the entire axial distance between the source and transducer. This has important implications in practice, as the water amplitudes are determined experimentally, and a reduced axial interval of integration implies less measurement time. The axial lengths over which amplitude data was required, in order to compute acoustic pressure and temperature fields accurate to a small tolerance (less than 10% difference with direct tissue simulations), were determined computationally for different transducer gains.

C. Results

In Fig. 1, contours of acoustic pressure error [Eq. (11a)] are plotted as a function of $\varepsilon$ and $N_w/N_r$, for gains of 20 [Fig. 1(a)], 40 [Fig. 1(b)], and 60 [Fig. 1(c)]. The contours may be interpreted using the following example. For a gain of 40, suppose experiments are performed in water at a reduced source acoustic pressure (relative to the desired source acoustic pressure in tissue) such that with $N_w = 0.61 N_r$ and the measured acoustic pressure amplitudes are reduced using the tissue absorption scaled down by a factor of 0.38. Then, the resulting derated acoustic pressure amplitudes will approximate those that would be measured directly in tissue within a 5% error. The acoustic pressure waveform in tissue, derived from the nonlinear derating process for $(N_w/N_r, \varepsilon) = (0.61, 0.38)$, is shown in Fig. 2. The acoustic pressure waveform generated through a direct simulation in tissue is also shown. The waveform produced by the derating technique is slightly broader than that generated by direct simulation, and the peak positive and negative acoustic pressures are quite similar.

As the gain increases, the bands in the contours of Fig. 1 shift toward higher values of $\varepsilon$, implying that an increased level of endpoint reduction is required as the level of focusing increases. The error bands also become more vertical as the gain increases, meaning that a wider range of source acoustic pressures in water can accurately produce the desired acoustic pressure field in tissue, provided that the absorption value is properly selected.

Contours of temperature error (Fig. 3) are very similar to those of acoustic pressure. For the example above where $(N_w/N_r, \varepsilon) = (0.61, 0.38)$, the expected error between the measured temperature in tissue and that predicted from nonlinear derating is again less than 5%. Temperature as a function of time is plotted in Fig. 4 for this example, for both the derating approach and direct simulation of temperature rise in tissue. At the end of sonication, the difference in temperature rise predicted by the derating technique and direct tissue simulation is about 1°C.

The contours in Fig. 3 are slightly less vertical than those in Fig. 1 for acoustic pressure, i.e., for a given source acoustic pressure in water ($N_w/N_r$ value), a wider range of absorption values can be used in the endpoint derating to generate the correct tissue temperature to a given level of accuracy than pressure. The thickness of the contours is also less for temperature, implying that the error increases more rapidly for temperature as one move away from optimal derating conditions.

The large amount of information contained in the contours of Figs. 1 and 3 can be distilled to a more practical form by considering a single value of $N_w/N_r$ and plotting error as a function of the sole parameter $\varepsilon$. We consider the case of $N_w/N_r = 0.7$, i.e., derating experiments are performed in water under conditions where the coefficient of


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nonlinearity is 70% of the value in tissue. In Fig. 5(a), the error (relative to values computed by direct simulation in tissue) for the derated acoustic pressure values is plotted as a function of $\varepsilon$ for this case. It can be seen that, for a gain of 20, the value of $\varepsilon$ that yields the minimum error for acoustic pressure is approximately 0.2. Thus, to complete the derating process (initiated by performing source scaling with $N_w = 0.7 N_t$), endpoint scaling should be performed by reducing the attenuation of each mode to 20% of its value appropriate for the tissue of interest at the given frequency. For a gain of 40, the attenuation of each mode should be reduced to approximately 42% of its non-derated value. The factor for a gain of 60 is about 61%. The minimum errors for the derated temperatures [Fig. 5(b)] occur when $\varepsilon = 0.18$.

FIG. 1. (Color online) (a) Contours of relative difference between focal acoustic pressure predicted by the derating technique and by direct simulation in tissue, as a function of the parameters $N_w/N_t$ and $\varepsilon$. $G = 20$. Coefficient of Nonlinearity in tissue $N_t = 0.3$. (b) Contours of relative difference between focal acoustic pressure predicted by the derating technique and by direct simulation in tissue, as a function of the parameters $N_w/N_t$ and $\varepsilon$. $G = 40$. Coefficient of Nonlinearity in tissue $N_t = 0.3$. (c) Contours of relative difference between focal acoustic pressure predicted by the derating technique and by direct simulation in tissue, as a function of the parameters $N_w/N_t$ and $\varepsilon$. $G = 60$. Coefficient of Nonlinearity in tissue $N_t = 0.3$. 


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FIG. 2. (Color online) Pressure waveform computed by derating method with $N_w/N_t = 0.61$, $\varepsilon = 0.38$, along with acoustic pressure waveform produced by direct simulation in tissue. $G = 40$. Coefficient of Nonlinearity in tissue $N_t = 0.3$.

$(G = 20), 0.42 (G = 40)$, and $1.0 (G = 60)$. Thus, to attain the minimum error in temperature, full attenuation should be used for each mode. For the highest gain $(G = 60)$, the optimal scaling factor $\varepsilon$ for the attenuation is considerably different for acoustic pressure and temperature. We note that it is not necessary to use the same value of $\varepsilon$ for both acoustic pressure and temperature computations. However, for simplicity, it can be useful to do so. For the gain of 60, a compromise between acoustic pressure and temperature accuracy could produce an $\varepsilon$ value around 0.7. This value yields both acoustic pressure and temperature errors less than 10%.

Regarding derating using reduced data sets, computations revealed that agreement with direct simulations could be maintained with less than 10% difference when the first 80% of the data in the axial direction ($0 < \zeta < 0.8$) was ignored. In Figs. 6(a) and 6(b), the focal waveform and temperature rise are shown for a case where the gain is 40 and $(N_w/N_t, \varepsilon) = (0.61, 0.38)$. In comparison with full simulation in tissue, reduced data set derating resulted in a decrease in accuracy from 3% to 4% for acoustic pressure, and from 2% to 9% for temperature.

IV. DISCUSSION

The premise of the derating strategy presented in this paper is that the nonlinear coupling effects in tissue can be determined from water measurements, provided that the pressure field is reduced (more precisely, the dimensionless parameter $N_t$ is reduced) at the source by the proper amount (scale factor $N_w/N_t$) when the water experiments are conducted. This “proper amount” can only be determined through a large battery of numerical experiments; we performed these calculations and computed the resulting error (relative to the standard of direct simulations in tissue). If the source is not reduced, the coupling effects are overpredicted. This source reduction $N_w/N_t$ alone, because it allows the complex nonlinear interactions in tissue to be computed from measurements in water, leads to reasonably accurate tissue pressures and temperatures. However, better accuracy is attained by further reducing the modal amplitudes. Since the pressures were already reduced at the source, reduction at the target location using the full attenuation is too severe. The parameter $\varepsilon$ was introduced to quantify how much of the full attenuation is required. [Each mode is attenuated a different amount, by the factor $\exp(-\varepsilon z_n z)$, as described in Sec. III A.] In terms of attaining minimum error defined by the metrics in Eqs. (11a) and (11b), the choices of $N_w/N_t$ and $\varepsilon$ are not unique. That is, for any given set of experimental conditions, there are multiple choices of $N_w/N_t$ and $\varepsilon$ that will yield the same errors (relative to simulations in tissue or actual measurements in tissue) in maximum and minimum pressure and maximum temperature. However, even though these errors are the same for different $(N_w/N_t, \varepsilon)$ combinations, other features of the pressure waveform will vary. These include RMS error over the whole waveform (which is related to the distribution of energy between harmonics), or just the peak negative pressure. So if a user happened to be particularly interested in accurately predicting peak negative pressure, say for determining the likelihood of cavitation, the user would want to select from the multiple $(N_w/N_t, \varepsilon)$ combinations the one that most accurately predicted peak negative pressure. Further numerical experiments would be required to identify this particular combination, as error in peak negative pressure was not a metric that was thoroughly explored in our study.

In practice, the error contours in Figs. 1 and 3 would be used to transform pressure measurements in water to focal pressure and temperature in tissue using the following sequence of steps. We illustrate using the specific example of a 1.98 MHz, 5.0 cm diameter transducer with a 6.26 cm focal length. The transducer acoustic power in tissue is approximately 42 W.

1. The transducer gain $G$ and nonlinear parameter $N_t$ are computed, using the transducer properties, the desired source acoustic pressure, and properties of the tissue medium. For the present case of interest, the gain $G = 40$ and $N_t = 0.3$. (Acoustic pressure at transducer surface = 0.34 MPa.)

2. We suppose that it is desired to obtain an error relative to direct simulations in tissue of less than 5%, for both pressure and temperature. From the contours in Figs. 1(b) and 3(b), which pertain to a gain of 40, we search for values of $N_w/N_t$ and $\varepsilon$ that will yield errors of less than 5% for both pressure and temperature. From Fig. 1(b), it can be seen that a pressure error of less than 5% can only be attained with an $\varepsilon$ value less than about 0.5 and a value of $N_w/N_t$ less than about 0.9. The choice $(N_w/N_t, \varepsilon) = (0.7, 0.47)$ resides roughly at the center of the 5% error bands in both Figs. 1(b) and 3(b), and we make this selection. If the transducer gain $G$ is significantly different from the values (20, 40, and 60) considered in Figs. 1 and 3, interpolation (or extrapolation) of the selected values of $N_w/N_t$ and $\varepsilon$ can be performed, or the computations described in Sec. II can be performed using the exact gain desired.

3. Experiments in water are performed with the source acoustic pressure set so that $N_w = N_t(N_w/N_t)$. In the present example, $N_w = 0.3 \times 0.7 = 0.21$. Pressure traces are recorded at axial locations spanning about the last 20% or 30% of the distance between the transducer and the
focus. For the present example, simulated pressures in water at the axial locations $z = 6.26 \text{ cm}$ and $6.0 \text{ cm}$, corresponding to the focus and a slightly pre-focal location, are shown in Figs. 7(a) and 7(b).

(4) In the focal plane, the acoustic pressure trace in water is also recorded at a radial location approximately $0.4 \text{ mm}$ off axis. The off-axis pressure for the present location is shown in Fig. 8.
Fourier transforms of the acoustic pressure vs time data are performed to obtain the source-derated modal amplitudes in water. Both real and imaginary parts are retained. The Fourier transform (real and imaginary parts) of the pressure measured at axial locations $z = 6.26$ cm and $6.0$ cm are shown in Figs. 9(a) and 9(b).

It can be seen that on the order of 10 harmonics are significant at the focus, and roughly half that many at the pre-focal locations. We emphasize that these quantities represent only source-derated water amplitudes, and not represent the amplitude estimates $w_n$, because they have not yet been endpoint derated.

The water modal amplitudes are then endpoint derated by multiplying by $\exp(-n\epsilon z)$. The resulting amplitudes are the fully derated water amplitudes, $w_n$. The $w_n$ values for the present example at the focal location are provided in Fig. 10. [The $w_n$ distribution at the pre-focal position (not shown) is similar to that in Fig. 10, though with less energy in the higher ($n > 5$) harmonics.] Endpoint derating has reduced the amplitudes to the point where only about half as many harmonics are now significant compared to the case of source-derating only [Fig. 9(a)].
The derated amplitudes $w_n$ are used to generate the modal products and sums in Eq. (8), resulting in an estimate for $q_n(f)$. The source term $q(f)$ is used in the numerical integral of Eq. (10) to obtain the final estimate for the tissue modal amplitudes, $a_n$. The $a_n$ values for the present example are provided in Fig. 11. [z = 6.26 cm. Pre-focal values (not shown) are similar but with little energy above harmonic 4.] Relative to the derated water amplitudes $w_n$ (Fig. 10), there has been a slight shift in energy toward the higher harmonics. For example, comparing $w_n$ to $a_n$ at the focus, the fundamental mode has decreased about 30% while the third harmonic has increased more than 20% (Figs. 10 and 11). The predictions for $a_n$ for the current example are compared with values derived through direct simulation in tissue in Fig. 12.

The modal amplitudes are used in (7) to compute the tissue acoustic pressure, and (6) to determine temperature rise. The off-axis measurements in water would be used in (6b) to determine the beam widths required for the temperature computation. For the present example, the focal pressure and temperature rise predicted by the derating technique agree with direct simulations in tissue within 10%, in line with the agreement between the modal amplitudes in Fig. 12.

We remind the reader that the data plotted in the contours all apply to the case $N_t = 0.3$. Contours for other $N_t$...
values can be obtained from the authors. Alternatively, the interested reader can perform the computations necessary to solve Eqs. (5)–(10) for the \( N_t \) value of interest (and, as noted above, the exact \( G \) value of interest.) The computations involving different \( (N_w/N_t, \epsilon) \) combinations can be terminated when an acceptable level of accuracy (relative to a tissue simulation for the prescribed \( N_t, G \) values) is obtained. The computation time required to simulate propagation in water, use the water modes to solve Eqs. (8)–(10), and compare with the reference field comprised of direct simulation in tissue, is about 2 h on a 4-core personal computer with 24 GB memory. As simulations using different \( (N_w/N_t, \epsilon) \) combinations are performed, the comparison simulation in tissue need not be redone.

In step 3 above, the amount of data acquisition necessary is dependent upon the transducer gain. The results of Fig. 6 show that an error of less than 10% in both acoustic pressure and temperature (relative to direct simulation in tissue) is possible when acoustic pressure measurements are made only in the last 20% of the axial locations \( (0.8 < \zeta < 1) \), when the transducer gain was 40. Smaller gains require larger data sets; approximately the last 25% is recommended for \( G = 20 \). Reduced data sets for very low gains \( (G < 10) \) were not explored; measurement over the entire axial range can be performed to ensure accurate results. For gains larger than 40, data sets smaller than the last 20% may suffice, though data sets less than 20% of the full amount were not examined by the authors.

From a computational standpoint, implementation of the method is straightforward, and requires little knowledge of numerical analysis by the user. Step 7 above involves algebraic manipulations of the derated modal amplitudes. Step 8 requires numerical quadrature, but the integrand is well behaved and simple methods suffice. As noted above, the trapezoidal rule was used in our computations.

The accuracy of the derating method was comparable at all gain values studied, in the sense that \( (N_w/N_t, \epsilon) \) combinations can be found at all gain values where good accuracy (<10% difference with tissue simulations) was achievable. As an example in the low-gain regime, for a set of computations performed with \( G = 10 \) and \( (N_w/N_t, \epsilon) = (0.79, 0.07) \), the acoustic pressure and temperature errors were both determined to be 7%. In the sense that the space of \( (N_w/N_t, \epsilon) \) values for which the error is less than 10% increases with increasing gain, the method is more accurate at higher gain values.

Besides applicability at low gains, an advantage of using the present approach to nonlinear derating is the direct availability of temperature estimates once the derated acoustic pressure amplitudes are determined. The modal beam widths are calculated from the off-axis acoustic pressure measurements and Eq. (6b), at which point Eq. (6a) provides the focal temperature as a function of time. For \( N_t \) values on the order of 0.3, less than 10 modes typically contribute to the temperature sum.

The Gaussian-mode approach becomes less accurate as the propagation becomes strongly nonlinear (Soneson and Myers 2007). The KZK approach, upon which the contours of Figs. 1 and 3 are based, also degrades in accuracy as the acoustic waveform becomes shocked, unless the model is
modified to treat shock formation and dissipation (Curra et al. 2000). Hence, the formulation presented in this paper is limited to moderately nonlinear procedures; we recommend an upper limit for the nonlinearity parameter in tissue of $N_t = 0.3$. Using typical transducer characteristics and tissue properties, this value corresponds to a power level of about 180 W when the gain is 40.

Full validation of the derating technique presented in this paper requires, in addition to the above comparisons with numerical simulations in tissue, comparisons with experiments performed in tissue or a tissue phantom. These experiments are underway. Experimental challenges include selecting the proper value of $r$, such that a sufficient number of radial modes can be resolved to make accurate temperature calculations. Accuracy (at the expense of increased measurement time) can be enhanced by making measurements at multiple radial locations, and curve-fitting higher-order modes using multiple data points. Averaging over the width of the hydrophone may also prove problematic for higher harmonics. This issue is minimized when propagation is limited to the moderately nonlinear propagation regime identified above, where less than 10 harmonics contribute.

V. CONCLUSION

A nonlinear-derating technique based upon Gaussian-mode theory proved to be a promising approach for characterizing HIFU systems. The technique was able to convert simulated acoustic pressure measurements in water to acoustic pressures and temperatures in tissue, with less than a 10% error when compared to results generated by direct numerical simulations in tissue. No assumption of Gaussian shading is required in the final results. Advantages of the derating approach include applicability over a wide range of gains, ease of computation (a single numerical quadrature is required), and readily obtained temperature estimates from the water measurements.


