Peristaltic transport of non-Newtonian fluid in a diverging tube with different wave forms

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Abstract

The present study investigates the peristaltic transport of non-Newtonian fluid, modeled as power law and Bingham fluid, in a diverging tube with different wall wave forms: sinusoidal, multi-sinusoidal, triangular, trapezoidal and square waves. Fourier series is employed to get the expressions for temporal and spatial dependent wall shapes. Solutions for time average pressure rise — flow rate relationship are computed for different amplitude ratios, $\varphi$, power law indices, $n$, yield stresses, $\tau_0$, and wave shapes. Results indicate that $\varphi$ and $n$ play a vital role in peristalsis. When $\varphi$ of the sinusoidal wave is increased from 0.6 to 0.8, the maximum pressure rise, $\Delta P_{L,\text{max}}$, increased by a factor of 10. Increasing $n$ from 0.6 to 1 increased the $\Delta P_{L,\text{max}}$ by a factor of 3. For Bingham fluid with $\varphi = 0.5$, a 25\% increase in $\Delta P_{L,\text{max}}$ is obtained when $\tau_0$, is reduced from 1 (non-Newtonian) to 0 (Newtonian). Of all the wave shapes considered, $\Delta P_{L,\text{max}}$ obtained is maximum for the square wave and minimum for the triangular wave (4–15 times less depending on $\varphi$). Finally, pathlines of massless particles are traced to investigate the occurrence of reflux. It is observed that, even for zero flow rate, reflux occurs near the tube wall and the thickness and shape of the reflux region strongly depends on $\varphi$, $n$, and shape of the peristaltic waves.

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1. Introduction

Peristaltic pumping is a form of physiological fluid transport that occurs in the human body. Peristaltic action is an inherent neuromuscular property of any tubular smooth muscle structure \cite{1}. During peristalsis, the fluid is driven by periodic progressive waves of contraction and expansion advancing axially along the distensible tube length. This mechanism is responsible for the transport of biological fluids in several physiological processes such as passage of urine from the kidneys to the bladder, the movement of chime in the gastrointestinal tract, transport of food bolus through the esophagus, transport of blood in small blood vessels, embryo transport in non-pregnant uterus,
and movement of spermatozoa in human reproductive tract. Improper or flawed peristalsis can cause pathological transport of bacteria, thrombus formation of blood [2], and infertility in human uterus [3,4]. This mechanism is also used for the design of pumps that propels fluid in situations where it is not desirable to have a direct contact between the fluid and the parts of the pumps, e.g. transportation of corrosive fluids.

Occurrence of peristalsis in different parts of the human body has been studied in recent past. Peristalsis in male reproductive system was observed experimentally and numerically by Batra [5], Guha et al. [6], Gupta et al. [7], and Srivastava et al. [8]. Srivastava et al. [8] modeled the peristaltic flow in the vas deferens by assuming it to be a non-uniform diverging channel and a tube. They looked at a more realistic model by evaluating non-Newtonian (power law fluid) fluid flow in a non-uniform tube. Li et al. [9], Misra et al. [10,11] studied, both experimentally and analytically, the movement of food bolus in the esophagus and gastrointestinal tract. Eytan et al. [3,4] investigated the effect of peristalsis in embryo transport within the uterine cavity. They discussed in detail the phenomenon of trapping and how the particle reflux impedes the embryo implantation at the fundus. Srivastava et al. [12] modeled blood as a casson fluid flowing inside small capillaries and blood vessels. Several literatures are available that encompass the mathematics of peristalsis and its effect on the fluid flow.

The essence of studying the peristaltic flow lies in solving the momentum equations. Several authors have attempted to solve these equations with diverse approximations applicable to their real physiological problem. A detailed summary of the theoretical investigations with the appropriate assumptions was presented by Srivastava et al. [12].

<table>
<thead>
<tr>
<th>Nomenclature</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>Radial coordinate</td>
</tr>
<tr>
<td>x</td>
<td>Axial coordinate</td>
</tr>
<tr>
<td>L</td>
<td>Length of the tube, m</td>
</tr>
<tr>
<td>a₀</td>
<td>Radius of tube at inlet, m</td>
</tr>
<tr>
<td>a(x)</td>
<td>Radius of tube at any distance x from inlet, m</td>
</tr>
<tr>
<td>μ</td>
<td>Newtonian viscosity, N s/m²</td>
</tr>
<tr>
<td>λ</td>
<td>Wavelength of the peristaltic wave, m</td>
</tr>
<tr>
<td>c</td>
<td>Wave speed, m/s</td>
</tr>
<tr>
<td>b</td>
<td>Amplitude of the peristaltic wave, m</td>
</tr>
<tr>
<td>p</td>
<td>Dimensionless pressure</td>
</tr>
<tr>
<td>τₗ,r′,x′</td>
<td>Shear stress normal to r′ in the x′ direction, N/m²</td>
</tr>
<tr>
<td>K</td>
<td>Divergence ratio</td>
</tr>
<tr>
<td>n</td>
<td>Power law index</td>
</tr>
<tr>
<td>k</td>
<td>Constant: (n/(3n + 1))ⁿ</td>
</tr>
<tr>
<td>y</td>
<td>Flow consistency</td>
</tr>
<tr>
<td>t</td>
<td>Dimensionless time</td>
</tr>
<tr>
<td>ϕ</td>
<td>Amplitude ratio (b/a₀)</td>
</tr>
<tr>
<td>H(x′, t′)</td>
<td>Wall coordinates, m</td>
</tr>
<tr>
<td>h</td>
<td>Dimensionless wall coordinate</td>
</tr>
<tr>
<td>u</td>
<td>Dimensionless velocity</td>
</tr>
<tr>
<td>Q</td>
<td>Dimensionless volume flow rate</td>
</tr>
<tr>
<td>ΔPₗ(t)</td>
<td>Dimensionless pressure rise (P_{exit}(t) − P_{in}(t)) along the tube length</td>
</tr>
<tr>
<td>ΔPₗ,max(t)</td>
<td>Dimensionless maximum pressure rise along the tube length</td>
</tr>
<tr>
<td>ΔPₗ,critical</td>
<td>Dimensionless time averaged critical pressure rise along the tube length</td>
</tr>
<tr>
<td>Q</td>
<td>Dimensionless time averaged flow rate</td>
</tr>
<tr>
<td>η</td>
<td>Coefficient of rigidity</td>
</tr>
<tr>
<td>τ₀</td>
<td>Yield stress, minimum shear stress to displace the fluid, N/m²</td>
</tr>
<tr>
<td>rₚ</td>
<td>Plug radius</td>
</tr>
<tr>
<td>uₚ</td>
<td>Plug velocity</td>
</tr>
<tr>
<td>Aₙ &amp; Bₙ</td>
<td>Fourier constants</td>
</tr>
</tbody>
</table>
Table 1
Summary of key assumptions made by recent literatures related to peristaltic transport

<table>
<thead>
<tr>
<th>Author</th>
<th>Assumptions</th>
<th>Re</th>
<th>Wave number</th>
<th>Fluid property</th>
<th>Flow geometry</th>
<th>Wave shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eytan et al. [4]</td>
<td>0</td>
<td>0</td>
<td>Newtonian</td>
<td>Two dimensional channel of varying cross-section</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Misra et al. [10]</td>
<td>Small</td>
<td>Small</td>
<td>Newtonian</td>
<td>Axi-symmetric tube of varying cross-section</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Eytan et al. [3]</td>
<td>0</td>
<td>0</td>
<td>Newtonian</td>
<td>Two dimensional channel of uniform cross-section</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Pozrikidis [2]</td>
<td>0</td>
<td>0</td>
<td>Newtonian</td>
<td>Uniform axi-symmetric tube</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Li et al. [9]</td>
<td>0</td>
<td>0</td>
<td>Newtonian</td>
<td>Uniform two dimensional tube</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Brasseur et al. [15]</td>
<td>0</td>
<td>0</td>
<td>Newtonian</td>
<td>Uniform two dimensional tube</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Misra et al. [11]</td>
<td>0</td>
<td>0</td>
<td>Power Law Fluid</td>
<td>Uniform axi-symmetric tube</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Mernone et al. [16]</td>
<td>Arbitrary</td>
<td>Arbitrary</td>
<td>Casson fluid</td>
<td>Uniform two dimensional channel</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Shakla et al. [17]</td>
<td>0</td>
<td>0</td>
<td>Power Law Fluid</td>
<td>Uniform axi-symmetric tube</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>El-Misery et al. [18]</td>
<td>0</td>
<td>0</td>
<td>Carreau fluid</td>
<td>Uniform two dimensional channel</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>El-Shewaiey et al. [19]</td>
<td>Small</td>
<td>Small</td>
<td>magneto fluid</td>
<td>Two dimensional channel</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Srinivas et al. [20]</td>
<td>0</td>
<td>0</td>
<td>Viscoelastic fluid</td>
<td>–</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Siddique et al. [21]</td>
<td>0</td>
<td>0</td>
<td>Third order fluid</td>
<td>Two dimensional channel</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Usha et al. [22]</td>
<td>0</td>
<td>0</td>
<td>Power Law Fluid</td>
<td>Two-layered aXi-symmetric tube</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Mekheimer [25]</td>
<td>0</td>
<td>0</td>
<td>Couple stress fluid</td>
<td>Two dimensional channel</td>
<td>Sinusoidal</td>
<td></td>
</tr>
<tr>
<td>Rao et al. [26]</td>
<td>0</td>
<td>0</td>
<td>Power Law fluid</td>
<td>Axi-symmetric porous tube</td>
<td>Sinusoidal</td>
<td></td>
</tr>
</tbody>
</table>

Table 1 gives an up to date summary of the literatures available in relevance to this topic and the list of approximations adopted by the authors.

This study, in many ways, improves upon Srivastava et al. [8] and other earlier assumptions, stated in Table 1, to enable more accurate analysis of the physiological and the pathophysiological phenomenon occurring during peristalsis. First, a non-uniform aXi-symmetric distensible tube is considered. This is because, most of the biological conduits are not strictly a channel or a tube of uniform cross-section. In particular, the vas deferens of a rhesus monkey is a diverging tube with a ratio of exit to inlet dimensions of approximately 4 [8].

Second, most of the literatures cited in Table 1 considered a Newtonian fluid for the analysis. However, from a realistic perspective, a food bolus passing through the esophagus, or urine flowing through the ureter, or chime traversing in the gastrointestinal tract, are essentially complex fluids that would not follow the Newtonian laws of viscosity. It would be more appropriate if the non-Newtonian behavior of the fluid is taken into account. Hence, in this study, the fluid is assumed to be non-Newtonian: power law and Bingham rather than Newtonian.

Third, the wall movement during peristalsis is modeled by considering limiting cases of wave shapes, i.e. from triangular to square waves in addition to the sinusoidal, multi-sinusoidal, and trapezoidal waves. Finally, the effects of non-Newtonian behavior of the fluid, amplitude ratio of the wall movement, and different wall shapes on the reflux phenomenon are studied in detail.

2. Formulation

2.1. Flow geometry

Non-Newtonian fluid (power law and Bingham) is considered to traverse along an aXi-symmetric tube of varying cross-section as depicted in Fig. 1. The tube has dimensions similar to that of the vas deferens of a rhesus monkey. The approximate values of various parameters in the vas deferens of rhesus monkeys, based on the experimental observations by Guha et al. [6], are

- Length of the tube, \( L = 20 \text{ cm} \)
- Inlet radius of the tube, \( a_0 = 0.012 \text{ cm} \)
- Outlet radius of the tube = 0.048 cm
- Newtonian viscosity of semen = 4 cp.

The ejaculation rate is about 0.02 ml/s for a pressure rise of 30 mmHg.
2.2. Assumptions

Assumptions made related to Reynolds number, periodicity of waves, wavelength, amplitude ratio, wave shape, and frame of reference for this study are:

* **Reynolds number:** Fluid flow inside the reproductive tract or gastrointestinal tract is essentially a creeping flow. Thus, the Reynolds number is very low (1 for ureter, 10 for gastrointestinal tract [1]). As a result, the inertia term, which is proportional to the square of velocity, can be neglected from the momentum equation in comparison to the linear viscous forces [27].

* **Periodicity of waves:** Li et al. [9] demonstrated that the overall performance of the peristaltic pump is not significantly altered by the non-integral numbers of peristaltic waves in a finite length tube. Accordingly, only periodic peristaltic waves in infinite tubes are considered and thus ignoring the unsteady effects associated with the finite length tubes.

* **Wavelength:** It may be noted that the radius of the vas deferens is very small \( a_0 = 0.012 \text{ cm} \) when compared to the wavelength \( L = \lambda = 20 \text{ cm} \) i.e. the wave number \( 2 \pi a / \lambda \) is generally small, which means that the theory of long wave length approximation is applicable here.

* **Amplitude ratio:** In comparison to the other parameters considered, the amplitude has the maximum influence on the flow rate. Therefore, more deliberation is required in choosing an optimal value for amplitude ratio, \( \varphi \), which is the ratio of amplitude of the peristaltic wave to the radius of the tube \( \varphi = b / a_0 \). Guha et al. [6] in their experimental observations estimated the value of \( \varphi \) to be 0.2. However, this is a purely rough estimate made during the non-ejaculatory state. Gupta et al. [7] assumed the value to be somewhere between 0.7 and 0.9 while Macht [14] suggested a larger value for \( \varphi \). No concrete experimental or theoretical reference is available in this regard. For the present investigation, a \( \varphi \) value between 0.5 and 0.8 has been chosen.

* **Wave shapes:** In addition to the sinusoidal waveform, the distensible tube wall is being assumed to have different wave shapes: multi-sinusoidal, trapezoidal, triangular, and square forms.

* **Frame of reference:** All the calculations are performed in the laboratory frame of reference and each particle along the tube wall travels in the radial direction only.

2.3. Mathematical formulation — Power law model

The \( r \) and \( x \) momentum equations are reduced to Stokes equation by applying lubrication theory. Moreover, the pressure is instantaneously uniform over each cross-section as wave number is very small (long wave length approximation). The appropriate Navier Stokes equations describing the flow in the laboratory frame of reference are

\[
\frac{\partial p'}{\partial x'} = \frac{1}{r'} \frac{\partial}{\partial r'} (r' \tau'_{r'x'})
\]

(1)

\[
\frac{\partial p'}{\partial r'} = 0
\]

(2)
where \( p', x', r' \) denotes pressure, axial, and radial coordinate, respectively, before non-dimensionalisation, and \( \tau_{r'x'} \) is the shear stress normal to \( r' \) in the \( x' \) direction. For power law fluid model, the \( \tau_{r'x'} \) is given as

\[
\tau_{r'x'} = y \left( \frac{\partial u'}{\partial r'} \right)^n
\]

with \( n \) being the flow behavior index and \( y(\frac{\partial u'}{\partial r'})^{n-1} \) is the apparent viscosity of the medium with \( y \) being the flow consistency index. For Newtonian fluid \((n = 1)\), \( y \) becomes the fluid viscosity.

The geometry of the wall surface (Fig. 1) is described as

\[
H(x', t') = a(x') + b \times \text{(Wave equation)}
\]

with

\[
a(x') = a_0 + K x'
\]

where \( a(x') \) is the radius of the tube at any axial distance from inlet, \( a_0 \) is the radius of the inlet, \( K \) is a constant whose magnitude depends on the length of the tube, exit and inlet dimensions, and \( b \) is the amplitude of the wave.

After the introduction of the dimensionless variables:

\[
\frac{r'}{a_0}, \quad \frac{x'}{\lambda}, \quad \frac{u'}{c}, \quad \frac{t'}{\lambda}, \quad \frac{p'}{p_0(a_0^{n+1}/y^c)}
\]

In view of Eq. (3), Eq. (1) assumes non-dimensional form as

\[
\frac{\partial p}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{\partial u}{\partial r} \right)^n \right]
\]

while a no-slip condition is imposed as a boundary condition for the \( u \) velocity, the particle is assumed to move only in the radial direction along the boundary.

\[
\frac{\partial u}{\partial r} = 0 \quad \text{at } r = 0
\]

\[
u = 0 \quad \text{at } r = h = 1 + \frac{\lambda K x}{a_0} + \varphi \text{ (Wave form)}
\]

where \( h = \frac{H}{a_0}, \varphi = \frac{b}{a_0} \)

Solving Eq. (7) and applying the boundary conditions, expression for velocity profile is:

\[
u(x, r, t) = n \left( -\frac{1}{2} \frac{\partial p}{\partial x} \right) \frac{1}{1 + n} \left[ h^{\frac{1+n}{n}} - r^{\frac{1+n}{n}} \right].
\]

The pressure rise \( \Delta P_L(t) \) in the tube of length \( L \), in the non-dimensional form, is obtained as [24]:

\[
\left( \frac{n}{3n+1} \right)^n \Delta P_L(t) = -2 \int_0^{\frac{L}{\lambda}} \left[ \frac{Q(x, t)}{h^{3n+1}} \right]^n dx.
\]

Expressions for \( h \) and \( Q(x, t) \) are derived in Sections 2.5 and 2.6, respectively.

2.4. Mathematical formulation — Bingham model

In case of Bingham fluid model, \( \tau_{r'x'} \) is given as

\[
\tau_{r'x'} = \eta \left( \frac{\partial u'}{\partial r'} \right) + \tau_0'
\]
where \( \eta \) is coefficient of rigidity and \( \tau'_0 \) is the minimum shear stress required to move the fluid. Considering Eq. (12), Eq. (1) assumes the non-dimensional form as

\[
\frac{\partial p}{\partial x} = \frac{1}{r} \frac{\partial}{\partial r} \left[ r \left( \frac{\partial u}{\partial r} + \tau_0 \right) \right]
\]

where \( \tau_0 = \tau'_0 (a_0 / \eta c) \) and other non-dimensional parameters are the same as that for the power law fluid. Applying the boundary conditions shown in Eqs. (8) and (9) and after integrating twice, the velocity profile is:

\[
u(x, r, t) = \frac{1}{4} \left( \frac{dp}{dx} \right) (r^2 - h^2) - \tau_0 (h - r).
\]

(14)

It should be noted that fluid does not move if

\[
\left( -\frac{r}{2} \right) \left( \frac{dp}{dx} \right) \leq \tau_0.
\]

(15)

Hence for \( r \leq r_p \), where \( r_p \) is the plug radius

\[
\frac{\partial u}{\partial r} = \frac{r}{2} \left( \frac{\partial p}{\partial x} \right) + \tau_0 \quad \text{i.e.} \quad r_p = \frac{2\tau_0}{\left( -\frac{dp}{dx} \right)}.
\]

(16)

But, for \( h > r > r_p \), \( u_p \) is:

\[
u_p = \frac{1}{4} \left\{ \left( -\frac{dp}{dx} \right) \left( h^2 - \left( \frac{2\tau_0}{-\frac{dp}{dx}} \right)^2 \right) \right\} - \tau_0 \left( h - \frac{2\tau_0}{-\frac{dp}{dx}} \right).
\]

(17)

The above equation can be simplified to get a relationship between \( Q(x, t) \) and \( \frac{dp}{dx} \) [23] as

\[
Z = -\frac{dp}{dx} = \frac{\left[ \frac{8Q(x, t)}{\pi} + \left( \frac{8\tau_0}{3} \right) h^3 \right]}{h^4}.
\]

(18)

2.5. Expression for wave shape

Five possible waveforms namely sinusoidal, multi-sinusoidal, triangular, trapezoidal and square waves are considered for the analysis. The flow geometry as seen in the laboratory frame of reference for all the five wave forms, is depicted in Fig. 2(a). As discussed earlier the dimensionless wall coordinates for any wave shape is given by the equation:

\[
h(x, t) = 1 + \frac{\lambda K x}{a_0} + \varphi \times \text{(Wave equation)}.
\]

(19)

Any wave shape equation may be obtained from Fourier series using the general equation:

\[
k + \sum_{n=1}^{\infty} A_n \cos(2n - 1)2\pi(x - t) + \sum_{n=1}^{\infty} B_n \sin(2n - 1)2\pi(x - t).
\]

(20)

Values of the Fourier constants \( k, A_n \) & \( B_n \) for the triangular, trapezoidal and square wave shapes are given in Table 2. The expressions for the wave forms derived from the Fourier series are given in Table 3.

The total number of terms in the Fourier series that are incorporated in the analysis is nineteen. Fig. 2(a) shows the position of wall at time \( t = 0 \) for different wave forms. Fig. 2(b) shows the change in position of the wall for a sinusoidal wave form as time is increased from \( t = 0 \) to \( t = 1 \). Since the wavelength of the traveling wave is equal to the tube length the wall profile at time \( t = 0 \) coincides with the profile at \( t = 1 \).
Table 2

Fourier constants for different wave forms

<table>
<thead>
<tr>
<th>Type of wave</th>
<th>( A_n )</th>
<th>( B_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangular</td>
<td>0</td>
<td>( \frac{8}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{(2n-1)^2} )</td>
</tr>
<tr>
<td>Trapezoidal</td>
<td>0</td>
<td>( \frac{32}{\pi^2} \sum_{n=1}^{\infty} \frac{\sin \frac{\pi}{8} (2n-1)}{(2n-1)^2} )</td>
</tr>
<tr>
<td>Square</td>
<td>( \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n+1}{(2n-1)^2} )</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3

Expressions for wall shape, \( h(x, t) \), for different wave forms

<table>
<thead>
<tr>
<th>Wave form</th>
<th>Expression for ( h(x, t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sinusoidal</td>
<td>( 1 + \frac{\lambda K x}{a_0} + \varphi \sin 2\pi(x - t) )</td>
</tr>
<tr>
<td>Multi-Sinusoidal</td>
<td>( 1 + \frac{\lambda K x}{a_0} + \varphi \left[ \frac{8}{\pi^3} \sum_{n=1}^{\infty} \frac{(-1)^n+1}{(2n-1)^2} \sin((2n-1)2\pi(x - t)) \right] )</td>
</tr>
<tr>
<td>Triangular</td>
<td>( 1 + \frac{\lambda K x}{a_0} + \varphi \left[ \frac{32}{\pi^2} \sum_{n=1}^{\infty} \frac{\sin \frac{\pi}{8} (2n-1)}{(2n-1)^2} \sin((2n-1)2\pi(x - t)) \right] )</td>
</tr>
<tr>
<td>Trapezoidal</td>
<td>( 1 + \frac{\lambda K x}{a_0} + \varphi \left[ \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n+1}{(2n-1)^2} \cos((2n-1)2\pi(x - t)) \right] )</td>
</tr>
</tbody>
</table>

Fig. 2. (a) Instantaneous wall location at time \( t = 0.9 \) for different waveforms. Straight lines depict the position of a wall before the start of peristalsis. (b) Change in wall shape with respect to time for sinusoidal wave form; wall profiles at time, \( t = 0 & 1 \) overlap with each other.

2.6. Instantaneous volume rate of flow in laboratory frame, \( Q(x, t) \)

The flow rates in the laboratory frame of reference, \( Q(x, t) \) and in the wave frame of reference, \( q(t) \) are related by the expression:

\[
\frac{Q(x, t)}{\pi} = \frac{q(t)}{\pi} + h^2.
\]  

Eq. (21)

Eqs. (11) and (21) can be combined to get

\[
\Delta P_L(t) = -2 \left( \frac{3n + 1}{n} \right)^n \left\{ q(t)^n \int_0^{\frac{L}{h^3n+1}} \frac{1}{h^{5n+1}} dx + \int \frac{1}{h^{5n-1}} dx \right\}.
\]  

Eq. (22)

Eq. (22) reduces to a simplified form of the equation obtained by Jaffrin et al. [1] for a Newtonian fluid of uniform cross-section.
Table 4
Expressions for instantaneous flow rate, \( Q(x, t) \), for different wave forms

<table>
<thead>
<tr>
<th>Wave form</th>
<th>Expression for ( Q(x, t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sinusoidal</td>
<td>( \frac{Q(x, t)}{\pi} = \frac{\bar{Q}}{\pi} - \frac{\varphi^2}{2} + (1 + \frac{\lambda K x}{a_0}) 2\varphi \sin 2\pi(x - t) + \varphi^2 \sin^2 2\pi(x - t) )</td>
</tr>
<tr>
<td>Multi-Sinusoidal</td>
<td>( \frac{Q(x, t)}{\pi} = \left[ \frac{\bar{Q}}{\pi} - \frac{\varphi^2}{2} + 2\varphi \left( 1 + \frac{\lambda K x}{a_0} \right) \sin \left( 2\pi m(x - t) \right) + \varphi^2 \sin^2 \left( 2\pi m(x - t) \right) \right] )</td>
</tr>
<tr>
<td>Triangular</td>
<td>( \frac{Q(x, t)}{\pi} = \left{ \begin{array}{ll} \frac{\bar{Q}}{\pi} - \frac{32\varphi^2}{\pi^4} \sum_{n=1}^{\infty} \left( \frac{1}{(2n-1)^4} \right) \ + \left( \frac{16\varphi}{\pi^2} \right) \left( 1 + \frac{\lambda K x}{a_0} \right) \sum_{n=1}^{\infty} \left( \frac{(-1)^{n+1}}{(2n-1)^2} \right) \sin \left( (2n-1)2\pi(x - t) \right) \ + \left( \frac{64}{\pi^4} \right) \varphi^2 \sum_{n=1}^{\infty} \left( \frac{1}{(2n-1)^4} \right) \sin^2 \left( (2n-1)2\pi(x - t) \right) \end{array} \right. )</td>
</tr>
<tr>
<td>Trapezoidal</td>
<td>( \frac{Q(x, t)}{\pi} = \left{ \begin{array}{ll} \frac{\bar{Q}}{\pi} - \frac{512\varphi^2}{\pi^4} \sum_{n=1}^{\infty} \left( \frac{\sin^2 \varphi}{\pi^2} \right) \left( \frac{2n-1}{2n-1} \right) \ + \left( \frac{64}{\pi^4} \right) \varphi^2 \sum_{n=1}^{\infty} \left( \frac{\sin^2 \varphi}{\pi^2} \right) \left( \frac{2n-1}{2n-1} \right) \sin^2 \left( (2n-1)2\pi(x - t) \right) \end{array} \right. )</td>
</tr>
<tr>
<td>Square</td>
<td>( \frac{Q(x, t)}{\pi} = \left{ \begin{array}{ll} \frac{\bar{Q}}{\pi} + \frac{8\varphi^2}{\pi^2} \sum_{n=1}^{\infty} \left( \frac{1}{(2n-1)^2} \right) \ + \left( \frac{8}{\pi^2} \right) \varphi^2 \sum_{n=1}^{\infty} \left( \frac{(-1)^{n+1}}{(2n-1)^2} \cos \left( (2n-1)2\pi(x - t) \right) \right) \ + \left( \frac{16}{\pi^2} \right) \varphi^2 \sum_{n=1}^{\infty} \left( \frac{1}{(2n-1)^2} \cos^2 \left( (2n-1)2\pi(x - t) \right) \right) \end{array} \right. )</td>
</tr>
</tbody>
</table>

In the above equation if both the integrations on the right-hand side are independent of time, then \( q(t) \) in wave frame becomes a constant (independent of time) provided \( \Delta P_L(t) \) is also a constant.

The time mean flow, \( \bar{Q}(x, t) \) over any wave period is given by:

\[
\frac{\bar{Q}}{\pi} = \frac{\varphi}{\pi} + \frac{1}{T} \int_0^T h^2 dt.
\]  \hspace{1cm} (23)

Here \( \bar{Q} = \frac{\bar{Q}}{a_0} \).

The instantaneous volume flow rate for sinusoidal wave form is obtained from combining Eq. (23) and equations in Table 3 as:

\[
\frac{Q(x, t)}{\pi} = \frac{\bar{Q}}{\pi} - \frac{\varphi^2}{2} + (1 + \frac{\lambda K x}{a_0}) 2\varphi \sin 2\pi(x - t) + \varphi^2 \sin^2 2\pi(x - t).
\]  \hspace{1cm} (24)

Expressions of instantaneous flow rate for all the wave forms are listed in Table 4.

In the above equations (Table 4), for \( n = 1 \) i.e. for Newtonian fluid, the expressions for \( h(x, t) \) and \( Q(x, t) \) reduces to same expressions obtained by Gupta et al. [7]. Further, for \( K = 0 \), an expression for a uniform tube can be derived, which corresponds to the results obtained by Jaffrin et al. [1]. Due to the complexity of the \( Q(x, t) \) expression the Eqs. (11) and (18) are not integrable analytically. Consequently, a numerical integration scheme is required for the evaluation of the integrals. A 2-point Newton–Cotes formula is used to evaluate the integral. MATLAB (Mathematics Laboratory) is used to evaluate the integrals and later to generate all the plots.
3. Results and discussions

Velocity profiles, temporal variation of pressure rise, $\Delta P_L(t)$, variation of time average pressure rise, $\overline{\Delta P}_L$, with time average flow rate, $\bar{Q}$, and pathlines are studied for both power law and Bingham fluids in the subsequent sections. Validations with previous studies [8] are shown wherever applicable.

3.1. Power law fluid

3.1.1. Velocity profile

The velocity profiles at different cross-sections for the power law fluid with $n = 1$ (Newtonian) are shown in Fig. 3. The figure also shows the positions of the distensible wall before and during the peristaltic motion (at time $t = 0.9$).

Fig. 3. Velocity profiles along the tube length for different wave forms at time, $t = 0.9$ and amplitude ratio, $\phi = 0.6$ (power law model).
Fig. 4. Validation of the present study: Temporal variations of pressure gradient \((k. \Delta P_L)\) along the tube length for power law indices, \(n = 1 \& 2/3\), time averaged flow rates, \(\overline{Q} = 0 \& 0.22\) and amplitude ratio, \(\varphi = 0.8\). Comparison between the present study and the study by Srivastava et al. (1985).

Fig. 5. Variation of average pressure rise \((k. \Delta P_L)\) with amplitude ratio \((\varphi)\) for sinusoidal, triangular and trapezoidal wave forms. \(k\) is given by the expression: \(k = \left(\frac{n}{3n+1}\right)^n\).

All the profiles are parabolic across the tube’s width and the fluid velocity alternates between positive and negative values due to the peristaltic motion. Maximum velocity magnitude is obtained at the narrowest cross-section, which occurs at the entrance of the tube for the sinusoidal, triangular and trapezoidal waves.

Since these are instantaneous plots, negative velocity profile does not imply that the net axial displacement over a peristaltic cycle is in the reverse direction. Magnitudes of the velocity profiles obtained depend on the amplitude ratio, \(\varphi\), of the traveling waves. When \(\varphi\) is increased from 0.6 to 0.8 (velocity profiles for \(\varphi = 0.8\) are not shown in Fig. 3), the fluid velocity rises by 15% and 10% for sinusoidal and triangular waves, respectively. This raise is more pronounced, i.e. about 2–3 times for the trapezoidal and square wave forms.

3.1.2. Validation

Validation for the present study is carried out by comparing maximum pressure rise, \(\Delta P_{L,\text{max}}\), obtained from the present study with the values obtained by Srivastava et al. [8] as shown in Fig. 4. For a Newtonian fluid, \(n = 1\), \(\Delta P_{L,\text{max}}\) obtained for the present study is 135.1 and 113.6 for the time average flow rates, \(\overline{Q}\), of 0 and 0.22, respectively. These values deviate from Srivastava et al. [8] by 0.36% and 0.84%, respectively. For a non-Newtonian
Fig. 6. Temporal variations of pressure gradient \((k.\Delta P_L)\) along the tube length for different wave forms with power law indices, \(n = 1, 0.8 \& 0.6\) and amplitude ratios, \(\varphi = 0.8, \& 0.6\). For the square wave form \((\Delta P_L)\) variation for \(\varphi = 0.6\) is not shown in the figure due to its relatively low value in comparison with \(\varphi = 0.8\).

fluid with \(n = 2/3\), \(\Delta P_{L,\max}\) obtained from the present study is within 9% of the values obtained by Srivastava et al. [8].

3.1.3. Effect of amplitude ratio, \(\varphi\), on time average pressure rise, \(\overline{\Delta P_L}\)

As discussed earlier, the values of amplitude ratio, \(\varphi\), used in this study are derived from previous literatures due to the lack of proper experimental data. However, a more realistic range for \(\varphi\) can be obtained by analyzing Fig. 5, which shows the variation of time average pressure rise, \(\overline{\Delta P_L}\), as a function of \(\varphi\). It is apparent from Fig. 5 that the \(\overline{\Delta P_L}\)
Fig. 7. Effect of power law index \((n)\) and amplitude ratio \((\varphi)\) on the average pressure rise \((\Delta P_L)\) - average flow rate \((\bar{Q})\) relationship for different wall shapes with power law index values, \(n\) ranging from 1 to 0.6 and amplitude ratios, \(\varphi\) from 0.6 to 0.8 (power law model).

Table 5
Comparison of the mean flow rate \((\bar{Q})\) obtained for different wave forms with the experimental data

<table>
<thead>
<tr>
<th>Model</th>
<th>Average flow rate in ml/s</th>
<th>Percentage error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experimental — [6]</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>Newtonian — [7]</td>
<td>0.007</td>
<td>66.3</td>
</tr>
<tr>
<td>Power law — [8]</td>
<td>0.011</td>
<td>45</td>
</tr>
<tr>
<td>Power law — Triangular</td>
<td>0.006</td>
<td>71.39</td>
</tr>
<tr>
<td>Power law — Trapezoidal</td>
<td>0.008</td>
<td>61.66</td>
</tr>
<tr>
<td>Power law — Square</td>
<td>0.009</td>
<td>57.25</td>
</tr>
</tbody>
</table>

values obtained are negligible for lower \(\varphi\) values and there is a significant increase in the \(\Delta P_L\) when \(\varphi\) exceeds 0.8. Since, only a small rise in \(\Delta P_L\) is expected across the tube length for physiological flow in vas deferens, a \(\varphi\) value below 0.8 is more realistic. Thus, an appropriate range for \(\varphi\) is between 0.5 and 0.8, which has been used in previous studies [8,13]. More experiments are required to support the above conclusion.

3.1.4. Temporal variation of pressure rise, \(\Delta P_L(t)\)

Temporal variations of pressure rise, \(\Delta P_L(t)\), for different power law indices, \(n = 0.6, 0.8,\) and 1, different amplitude ratios, \(\varphi = 0.6\) and 0.8, and different wave shapes are shown in Fig. 6. For a straight tube, \(\Delta P_L\) value
remains constant and does not change with time. However, it is apparent from Fig. 6 that for a diverging tube, $\Delta P_L(t)$ changes with time. $\Delta P_L(t)$ for a sinusoidal wave, shown in Fig. 6(a), increases with time and attains a maximum value at $t = 0.27$. At this time, maximum occlusion occurs at the tube entrance where the cross-sectional area is a minimum and consequently $\Delta P_L$ obtained is a maximum. Subsequently, $\Delta P_L(t)$ starts decreasing with time and drops back to zero at $t = 1$. Similar trend is observed for all the other wave forms. However, parameters like $n$, $\varphi$ and wave shape significantly affect the $\Delta P_{L,\max}$ value and the time at which it occurs.

**Effect of power law index, $n$:** It can be seen from Fig. 6 that $\Delta P_{L,\max}$ increases with increase in power law index, $n$. For a sinusoidal wave with $\varphi = 0.6$, $\Delta P_{L,\max}$ increases by $\sim 83\%$, when $n$ is increased from 0.6 to 0.8. When $n$ is increased from 0.8 to 1, $\Delta P_{L,\max}$ increases by $\sim 86\%$. This pattern of rise in $\Delta P_{L,\max}$ is augmented for larger amplitude waves. For $\varphi = 0.8$, $\Delta P_{L,\max}$ increases by 179% and 188%, respectively, when $n$ is increased from 0.6 to 0.8 and then to 1. Similar trend is observed for all the other wave forms as shown in Fig. 6.

**Effect of amplitude ratio, $\varphi$:** The most important parameter that influences the fluid flow during peristalsis is the amplitude ratio, $\varphi$, of the wall movement. It is evident from Fig. 6 that $\Delta P_{L}(t)$ values increase with increase in $\varphi$. For instance, a sinusoidal wave with $\varphi = 0.8$ produces a $\Delta P_{L,\max} \sim 10$ times higher than the $\Delta P_{L,\max}$ obtained for a wave with $\varphi = 0.6$. It is also observed that for triangular, multi-sinusoidal, and trapezoidal wave forms, $\Delta P_{L,\max}$ increases $\sim 10$ fold when $\varphi$ is increased from 0.6 to 0.8. However, for the square wave, $\Delta P_{L,\max}$ increases appreciably from 28.7 to 2804.6, for a similar increase in $\varphi$ value. This trend is observed even for lower values of $n$ (0.6 and 0.8) but with lesser magnitude of increase as compared to $n = 1$.

**Effect of different wave forms:** $\Delta P_{L,\max}$ values depend upon the degree of occlusion produced by the moving boundaries. Of all the wave forms discussed, only the square wave produces near complete occlusion as shown in Fig. 2(a). As a result, square wave produces the maximum $\Delta P_{L,\max}$ value. Fig. 2(a) also shows that the trapezoidal wave, whose shape resembles the square wave, produces the second highest degree of occlusion followed by sinusoidal and multi-sinusoidal waves. Triangular wave produces the lowest level of occlusion, and consequently results with minimum rise in $\Delta P_{L,\max}$. 

Fig. 8. (a) Trajectories of fluid particles for a sinusoidal wall profile; mean flow rate, $Q = 0$, power law index, $n = 1$, amplitude ratio, $\varphi = 0.5$; • Initial location of the particle. (b) Reflux boundary separating the forward and reverse flowing regions for a sinusoidal wall movement; $Q = 0$, power law index, $n = 1$, amplitude ratio, $\varphi = 0.5$. 
3.1.5. Effect of time average flow rate, $\bar{Q}$, on time average pressure rise, $\Delta P_L$

Fig. 7 shows the average pressure rise, $\Delta P_L$, as a function of time average flow rate, $\bar{Q}$, for the power law fluid. $\bar{Q}$ increases linearly with decrease in $\Delta P_L$ and minimum $\bar{Q}$ is obtained when $\Delta P_L$ is a maximum and vice versa.

In the absence of peristalsis i.e. $\varphi = 0$, flow resembles the Poiseuille flow through a diverging tube. However, during peristalsis, depending upon the degree of occlusion, a positive flow rate is obtained even for an adverse pressure gradient. The $\Delta P_L$ value for which $\bar{Q}$ becomes zero, termed as time average critical pressure rise ($\Delta P_{L,\text{critical}}$), is zero for a Poiseuille flow and is positive for a flow with peristalsis. The exact value of $\Delta P_{L,\text{critical}}$ depends on parameters like $n$, $\varphi$ and wave shape.

Fig. 7(a) shows $\Delta P_L$ as a function of $\bar{Q}$ for a sinusoidal wave with different $\varphi$ and $n$ values. Non-dimensional $\Delta P_{L,\text{critical}}$ value for a sinusoidal wave with $\varphi = 0.6$ and $n = 1$ is 3.1. Thus, peristalsis can overcome a pressure rise of 3.1 and still pump a Newtonian fluid in the forward direction. $\Delta P_{L,\text{critical}}$ increases with increase in $\varphi$. When $\varphi$ is increased from 0.6 to 0.8, $\Delta P_{L,\text{critical}}$ increases from 3.1 to 20.0 for a Newtonian fluid.

It can be observed from Fig. 7(a) that as $n$ decreases, the $\Delta P_{L,\text{critical}}$ value also decreases. For $\varphi = 0.8$, the ratio of $\Delta P_{L,\text{critical}}$ for a Newtonian fluid to a fluid with $n = 0.6$ is 16. This suggests that Newtonian fluid can overcome the pressure rise much better unlike non-Newtonian fluids, which are more prone to reverse flow. Fig. 7 also compares the pressure-flow characteristics for different waveforms. It is observed that, for a fluid with $n = 1$ and $\varphi = 0.8$, square wave has the maximum non-dimensional $\Delta P_{L,\text{critical}}$ value of 143.0 and triangular wave has the minimum value of 8.6. As a result, reverse flow starts much earlier for a triangular wave and is delayed for the square wave. Thus, square wave has the best pumping characteristics of all the wave forms whereas triangular wave has the worst pumping characteristics.

Table 5 compares the $\bar{Q}$ obtained for different wave forms with the experimental results obtained by Guha et al. [6]. Average flow rate through the vas deferens of a rhesus monkey was determined experimentally as 0.02 ml/s for a pressure rise of 30 mmHg. Srivastava et al. [8] obtained the value of $\bar{Q}$ as 0.011 ml/s for a power law fluid with sinusoidal wave and $\varphi = 0.9$. This $\bar{Q}$ value is much closer to the actual experimental value. However, from Fig. 4 the non-dimensional $\Delta P_L$ for $\varphi = 0.9$ would be around 200.0, which could be high for a physiological process.
Table 5 also shows that square wave produces the maximum $Q$ of 0.009 ml/s whereas the triangular wave results in a minimum $Q$ of 0.006 ml/s. All the other wave forms produce $Q$ that lie between these two limiting values.

3.1.6. Particle trajectory and reflux phenomenon

Occurrence of reflux can be examined by tracking the pathlines of the massless particles in the Lagrangian frame of reference during the time the peristaltic wave completes one full cycle, i.e. when wave travels through exactly the length of the tube. Pathlines can be traced by integrating the velocity components over time. Particle pathlines for different wave forms with $\varphi = 0.8$ are shown in Fig. 8 to 10 for two flow rates $Q = 0$ and $Q = 0.77$. Dotted lines in figures represent the position of the tube wall before peristalsis.

Pathlines traced in Fig. 8(a) indicate that even for a zero mean flow rate, $Q = 0$, particles near the axis of the tube undergo a net positive displacement. A massless particle located near the axis (at $x = 0.2, r = 0.2$) undergoes net positive displacement of 0.11 and 0.02 in axial and radial directions respectively. However, for the particles located near the exit of the tube, axial and radial movements are constrained. For example, a particle starting from $(0.4, 0.2)$ travels a distance of 0.48 (0.63 for a particle at 0.2, 0.2) and a particle at $(0.6, 0.2)$ travels an even lesser distance of 0.35 before coming to rest. This decline in the mobility of the particle can be attributed to the diverging nature of the tube, which gradually dilutes the effect of peristalsis as the tube cross-section increases. Thus, particles at the exit are less influenced by the peristalsis. However, the particles located near the tube wall follows a different pattern.

Occurrence of reflux can be traced near the wall, where the particles have a net axial displacement in the negative direction. For example, in Fig. 8(a) the net axial displacement of the particle near the axis (0.6, 0.2) is 0.19. However, for a particle near the wall (0.6, 2), net axial movement of the particle is in negative direction $(-0.05)$. This happens because, the axial velocity component decreases radially and the particles tend to move in the radial direction and the pathlines become aligned towards the transverse axis.

Based on the particle pathlines obtained from Fig. 8(a), thickness of the reflux region is calculated and the boundary separating the forward and reverse flowing regions is displayed in Fig. 8(b). It can be seen from the figure that the thickness and shape of the reflux zone depends significantly on the wall movement. Reflux boundary
varies as a function of axial and radial distance and takes the shape of the moving wall. Thickness of the region is calculated as 0.6 and is constant for the most part of the tube, except at the exit where the thickness is around 2.

Particle trajectories for other wall shapes are shown in Figs. 9 and 10. It is interesting to note from Fig. 9(a) that particle reflux starts occurring at a smaller radius for the multi-sinusoidal wave than for a sinusoidal wave and the magnitude is also relatively more for the multi-sinusoidal wave. Owing to its poor pumping characteristics, triangular wave constraints the movement of the particles. For trapezoidal and the square waves shown in Fig. 10(a) and (b), the particle paths at different locations overlap one another suggesting that the particles are involved in increased movement both axially and radially.
Fig. 12. Temporal variations of pressure gradient \( (k \Delta P_L) \) along the tube length for different wave forms with yield stress values, \( \tau_0 = 0 \) & 1 and amplitude ratios, \( \varphi = 0.5, 0.6 \) & 0.7.

3.2. Bingham fluid

3.2.1. Velocity profile

Fig. 11 shows the velocity profile for a Bingham fluid. The solid plug zone is evident in Fig. 11 where the applied shear stress is below the yield stress, \( \tau_0 \). Beyond this plug zone the shear stress is more than the \( \tau_0 \) and the fluid behaves like a Newtonian fluid. Velocity magnitudes follow a similar trend as observed in the power law fluid. As \( \tau_0 \) is increased from 0 to 1, the velocity decreases by \( \sim 7\% \) for sinusoidal, triangular and trapezoidal waves. For the square wave, a \( \sim 31\% \) drop in the velocity is observed.
Fig. 13. Dependence of average pressure rise \( (k \Delta P_L) \) — average flow rate \( \bar{Q} \) relationship on the amplitude ratio \( (\varphi) \) and yield stress \( (\tau_0) \) for different wall shapes: yield stress, \( \tau_0 = 0 \& 1 \) and amplitude ratios, \( \varphi = 0.5, 0.6 \& 0.7 \) (for Bingham fluid).

3.2.2. Temporal variation of pressure rise, \( \Delta P_L(t) \)

The pressure rise, \( \Delta P_L \), is plotted as a function of time, \( t \), in Fig. 12 for \( \varphi = 0.5, 0.6, \) and 0.7 and for maximum and minimum yield stresses \( (\tau_0 = 0 \& 1) \). Fig. 12(a) shows that, \( \Delta P_{L_{\text{max}}} \) decreases with increase in \( \tau_0 \). For \( \varphi = 0.5 \), increasing \( \tau_0 \) from 0 to 1 causes the \( \Delta P_{L_{\text{max}}} \) to decrease by \( \sim 25\% \), \( \sim 15\% \), and \( \sim 15\% \) for sinusoidal, trapezoidal, and square waves, respectively. However, for a higher \( \varphi \) value of 0.7, \( \Delta P_{L_{\text{max}}} \) drops only by \( \sim 5\% \), \( \sim 2.5\% \), and \( \sim 1.5\% \) for the above mentioned wave forms. Thus, for higher degree of occlusion i.e. large \( \varphi \) value, \( \Delta P_{L_{\text{max}}} \) produced by trapezoidal and square wave is not significantly changed by the yield stress, \( \tau_0 \), of the fluid. However, the same conclusion cannot be drawn in the case of the sinusoidal wave where \( \Delta P_{L_{\text{max}}} \) changes significantly with \( \tau_0 \).

3.2.3. Effect of time average flow rate, \( \bar{Q} \), on time average pressure rise, \( \bar{\Delta P}_L \)

\( \bar{Q} \) increases linearly with decrease in the \( \bar{\Delta P}_L \) value, similar to the power law fluid (Fig. 13). Dependence of \( \bar{\Delta P}_{L_{\text{critical}}} \) on \( \varphi \) follows the same trend as discussed in the power law fluid. It is also observed that, for trapezoidal and square waves, the non-Newtonian effects \( (\tau_0) \) of a Bingham fluid has lesser influence on the average pressure rise. For a sinusoidal wave with \( \varphi = 0.5 \), \( \bar{\Delta P}_{L_{\text{critical}}} \) decreases by \( \sim 83\% \) when \( \tau_0 \) is increased from 0 to 1. On the contrary, for a square wave with same \( \varphi \), \( \bar{\Delta P}_{L_{\text{critical}}} \) decreases only by \( \sim 23\% \) for the same rise in \( \tau_0 \) value. For a higher \( \varphi \) value of 0.7, \( \bar{\Delta P}_{L_{\text{critical}}} \) decreases by \( \sim 19\% \) for the sinusoidal wave while it decreases only by 4% for a square wave. In comparison to this, a significant increase in \( \bar{\Delta P}_L \) value is obtained for the power law fluid.
4. Conclusions

In this study, an attempt has been made to improve upon previous models representing peristaltic transport of biological fluid through a distensible tube. To begin with, lubrication theory was employed to study the peristaltic transport of non-Newtonian fluids: power law and Bingham fluids. Results suggest that the $\Delta P_L$ value has increased dependence on the amplitude ratio, $\varphi$, and power law index, $n$. It has been shown that non-Newtonian fluids are more prone to reverse flow than Newtonian fluids. Results also suggest that yield stress, $\tau_0$, has a considerable influence on the $\Delta P_L$ obtained. It would be further interesting to investigate the combined effect of yield stress and the power law index on the flow properties by employing the Herschel–Bulkley model for non-Newtonian fluid.

Pressure flow characteristics of various wave forms, in addition to the sinusoidal wave form, have been studied. The rationale behind assuming different wall shape was to bind the flow problem with the limiting cases of wall motion; triangular wave and square wave. Results obtained suggest that square wave has the best pumping characteristics of all the wave forms and triangular wave has the worst characteristics. However, results suggest that very high $\Delta P_L$ values are obtained for square wave, which may not represent a realistic model with physiological significance. The high flow rate, obtained for the square wave, still falls well short of available experimental value. This suggests that there are other factors, in addition to the peristalsis, that cause the fluid motion in the vas deferens. Nevertheless, the above discussion endorses the necessity for obtaining more experimental data and thus validating the peristaltic flow model more accurately.

Finally, particle trajectories have been traced to explore the occurrence of reflux. It has been observed that even under zero flow rate condition particles near the wall move upstream against the peristalsis and have a net negative displacement as suggested by Jaffrin et al. [1]. The occurrence of reflux has a strong dependence on the degree of occlusion, magnitude of $\Delta P_L$ across the tube, and shape of the peristaltic wave.

References


