Characterization of high intensity focused ultrasound transducers using acoustic streaming

Prasanna Hariharan  
Division of Solid and Fluid Mechanics, Center for Devices and Radiological Health, U. S. Food and Drug Administration, Silver Spring, Maryland 20993, USA and Mechanical, Industrial, and Nuclear Engineering Department, University of Cincinnati, Cincinnati, Ohio 45221, USA

Matthew R. Myers, Ronald A. Robinson, and Subha H. Maruvada  
Division of Solid and Fluid Mechanics, Center for Devices and Radiological Health, U. S. Food and Drug Administration, Silver Spring, Maryland 20993, USA

Jack Sliwa  
St. Jude Medical, AF Division, 240 Santa Ana Court, Sunnyvale, California 94085, USA

Rupak K. Banerjee  
Mechanical, Industrial and Nuclear Engineering Department and Biomedical Engineering Department, 598 Rhodes Hall, P.O. Box 210072, University of Cincinnati, Cincinnati, Ohio 45221, USA

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A new approach for characterizing high intensity focused ultrasound (HIFU) transducers is presented. The technique is based upon the acoustic streaming field generated by absorption of the HIFU beam in a liquid medium. The streaming field is quantified using digital particle image velocimetry, and a numerical algorithm is employed to compute the acoustic intensity field giving rise to the observed streaming field. The method as presented here is applicable to moderate intensity regimes, above the intensities which may be damaging to conventional hydrophones, but below the levels where nonlinear propagation effects are appreciable. Intensity fields and acoustic powers predicted using the streaming method were found to agree within 10% with measurements obtained using hydrophones and radiation force balances. Besides acoustic intensity fields, the streaming technique may be used to determine other important HIFU parameters, such as beam tilt angle or absorption of the propagation medium. © 2008 Acoustical Society of America. [DOI: 10.1121/1.2835662]

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I. INTRODUCTION

Most therapeutic ultrasound procedures, including tumor ablation, hemostasis, and gene activation, rely on the ability of high intensity focused ultrasound (HIFU) to rapidly elevate tissue temperatures. In order to maximize the effectiveness of the procedure, as well as to avoid collateral tissue damage, it is desirable to predict the energy distribution of the HIFU beam within the propagation medium. An important first step in this process is the characterization of the ultrasound beam in a liquid medium. In this characterization, the acoustic intensity is determined throughout the spatial volume of interest, for transducer power levels of practical importance.

Currently, HIFU fields are often characterized using radiation force balance and hydrophone techniques, which measure the ultrasonic power and intensity distribution, respectively (Shaw and ter Haar, 2006; Harris 2005). Although these two techniques are well established and widely used, there are known limitations to both of these methods, including: (i) sensor damage due to heating and cavitation; (ii) inaccuracies due to strong focusing; and (iii) inaccurate frequency response due to generation of higher harmonics (Shaw and ter Haar, 2006). Consequently, these techniques can accurately characterize HIFU transducers only at low power. For clinically relevant high powers, there are no alternative measurement standards available to accurately characterize medical ultrasound fields generated by HIFU transducers (Shaw and ter Haar, 2006; Harris 2005).

Several new methods for measuring HIFU fields are being researched, including development of robust sensors and hydrophones (Wang et al., 1999; Shaw, 2004; Schafer et al., 2006; Zanelli and Howard, 2006; Shaw and ter Haar, 2006). An alternative approach to overcome the sensor-induced inaccuracies is to eliminate the use of sensors, and noninvasively measure the pressure field. One such commercially available noninvasive method is the schlieren imaging technique (Harland et al., 2002; Theobald et al., 2004), which utilizes changes in the optical index of refraction to qualitatively define the ultrasound field. However, for quantitative evaluation, the pressure field must be reconstructed tomographically. Other than schlieren imaging, there are no noninvasive techniques reported in recent literature capable of measuring ultrasound field at high powers.

a)Author to whom correspondence should be addressed. Electronic mail: matthew.myers@fda.hhs.gov
This paper describes a noninvasive method that is capable of measuring the acoustic intensity in a free field. The method incorporates acoustic streaming, the steady fluid movement generated when propagating acoustic waves are attenuated by viscosity of the fluid medium (Nyborg, 1965). Acoustic streaming arising from ultrasound absorption was first discussed by Eckart (1948), who derived an expression for the streaming velocity by applying the method of successive approximations to the Navier-Stokes equations. According to Eckart’s theory, the acoustic streaming velocity is directly proportional to the square of acoustic pressure, and inversely proportional to the shear viscosity. However, Eckart’s expression was derived ignoring the hydrodynamic nonlinearity term in the Navier-Stokes equation. Later, Lighthill (1978a, b) established that this steady streaming motion is due to the mean momentum flux (Reynolds stress) created by the viscous dissipation of acoustic energy in the fluid medium. Subsequently, Starritt et al. (1989, 1991), Tjotta and Tjotta (1993), and Kamakura et al. (1995) investigated, both experimentally and numerically, the effect of acoustic and hydrodynamic nonlinearity on the streaming velocity for a focused ultrasound source. Results obtained from their experiments and computations suggest that both acoustic and hydrodynamic nonlinearities can play a major role in generation of acoustic streaming.

Previous studies have exploited the relationship between the acoustic streaming field and the ultrasound intensity to various degrees. Nowicki et al. (1998) used both particle image velocimetry (PIV) and Doppler ultrasound to measure streaming motion generated by a weakly focused ultrasound beam in a solution of water and cornstarch. They found that similar streaming velocities were obtained for both the methods and the velocity magnitude was directly proportional to the acoustic power emitted by the transducer. Hartley et al. (1997) and Shi et al. (2002) used Doppler ultrasound to quantitatively measure acoustic streaming velocity in blood when exposed to an ultrasound source. They used streaming as a potential tool for improving hemorrhage diagnosis. Hartley et al. (1997) found that the streaming velocity increases with acoustic power and reduces with increase in viscosity during blood coagulation. Choi et al. (2004) used PIV to characterize streaming motion induced by a lithotripter. They observed that the streaming velocity correlated linearly with the peak negative pressure of the acoustic field measured at the focus. More recently, Madelin et al. (2006) used MRI to measure streaming velocity in a Glycerol water mixture. From the streaming velocity they tried to estimate the fluid properties such as attenuation and bulk viscosity. They used the expression derived by Eckart (1948) to calculate the acoustic field and time-averaged acoustic power of a plane ultrasound transducer from the streaming velocity data. While all of these experimental studies have observed a correlation between the acoustic intensity field (and acoustic power) and the acoustic streaming field, none of them attempted to use this correlation to accurately predict the acoustic intensity field.

Our characterization technique employs a predictor-corrector type method that back calculates the total ultrasonic power and acoustic intensity field from the streaming velocity field generated by the HIFU transducers. The acoustic streaming field set up by the HIFU beam with unknown energy distribution is measured experimentally using digital particle image velocimetry ( DPIV ) (Prasad, 2000). Then, an initial guess for the unknown acoustic intensity field is made. Based upon the Reynolds stress derived from this intensity, the streaming velocity is computed and compared with the experimental values. Using the error between the computed and observed velocity fields, an optimization routine is employed to refine the guess for the intensity field and the computation procedure is repeated. When the difference between the computed and measured velocity fields falls below a threshold value, the final estimate for the ultrasound intensity field is obtained. This inverse method can also be used to estimate physical properties of the fluid medium such as ultrasound absorption coefficient, or properties of the beam such as tilt angle.

The acoustic intensities of interest in this paper are those that are low enough that nonlinear propagation may be neglected, yet high enough to produce temperature increases in tissue of a few degrees per second. We label this range the “moderate” temperature regime (Hariharan et al., 2007). An order-of-magnitude estimate of this regime is 100–1000 W/cm², with precise values depending upon the amount of focusing, nonlinear parameter of the tissue, etc. Despite the assumption of linear propagation, intensities in the moderate regime may be damaging to conventional hydrophones, and hence good candidates for measurement via the streaming method. In terms of transducer powers, the values considered were in the range 5–30 W.

The streaming technique is described in Sec. II. In Sec. III, the intensity field and acoustic power calculated using this approach are compared with measurements made using standard measurement techniques such as hydrophone scans and radiation force balance (RFB). The applicability of the technique is summarized in Sec. IV.

II. MATERIALS AND METHODS

The transducer-characterization method utilizes an optimization algorithm, in which the difference between the experimental streaming velocity and the computed velocity is minimized as a parameter (or parameter set) characterizing the acoustic field is varied. The parameter of interest is often the total power, though quantities such as the transducer focal length, transducer effective diameter, or medium attenuation may also be determined. A flow chart of the algorithm is shown in Fig. 1. The following sections describe the steps of the flow chart.

A. Experimental measurement of streaming velocity

Figure 2 shows the experimental apparatus. Our experimental setup combines two different systems—(i) acoustic streaming generation system [Fig. 2(a)] and (ii) DPIV measurement system (Integrated Design Tools, Tallahassee, FL) [Fig. 2(b)].

In the streaming generation system, the HIFU transducer to be characterized is mounted vertically in a Plexiglas™ tank using a triaxis positioning system [Fig. 2(a)]. Sound
sources considered in this study are single element, spherically focused, piezoceramic transducers with diameter ranging from 6.4 to 10.0 cm and focal length varying between 6.3 and 15.0 cm (Table I). The central frequency of the transducers ranges from 1.1 to 1.5 MHz. The driving signal for the transducer comes from a wave form generator (Wavetek 81, Fluke Corp., Everett, WA) in conjunction with an rf amplifier (Model 150A 100B, Amplifier Research, Souderton, PA). The output signal from the amplifier was monitored using an oscilloscope (Model 54615B, Agilent, Santa Clara, CA).

The measurement system employs a cubical Plexiglas tank (20 × 20 × 20 cm) filled with water or other appropriate fluid medium. The fluid medium was seeded with 10 μm hollow glass spheres (Dantec Dynamics Inc., Ramsey, NJ) which act as tracer particles for capturing the flow. The specific gravity of the spheres was approximately 1.1. Radiation force acting on the tracer particles was calculated to be negligible with respect to the measured streaming velocity (from DPIV system). This suggests that the tracer particles used in this study do not undergo motion relative to the streaming flow. In the experiments, particles were added until 5 to 10 particle pairs per interrogation area were obtained.

The DPIV system incorporates a dual pulsed 15 mJ/pulse Q-switched multimode Nd:YAG laser (λ =532 nm), with a beam diameter of 2.5 mm, repetition rate of 15 Hz, and a measured pulse width of 10 ± 1 ns, as the illumination source (New Wave Solo 1, New Wave Corp., Freemont, CA). A 100 mm focusing lens was used as laser-to-fiber coupler [Fig. 2(b)] for focusing the 2.5 mm laser on to the fiber face. The focal spot diameter at the fiber face was measured as 178 μm ± 1 μm.

Instead of solid silica-core fibers, which are not suitable for higher power delivery, this system incorporated a 700 μm core diameter hollow waveguide (HW) delivery fiber (Robinson and Ilev, 2004). This fiber was coated with cyclic olefin polymer to minimize the HW attenuation losses at the 532 nm wavelength. The output of the fiber was then directed through the microscope objective lens (4×). The laser beam was then passed through the final Powell lens optics, which produced a thin laser sheet for illumination of the flow model.

Flow visualization and recording were done using an 8 bit 1K×1K CCD camera (double exposure Kodak ES 1.0), which has a field of view of 2 cm×2 cm. The CCD camera was connected to a computer via frame grabber to store and process the recorded images. The computer also synchronizes the camera with the laser pulse source. The time between consecutive image pairs, which is the inverse of laser pulse repetition rate, was set at 1/15 s. The pulse delay time or the time between the images was varied between 1000 and 10 000 μs depending on the streaming velocity.

Measurement of the streaming field was initiated with the positioning of the axis of the HIFU beam within the laser sheet. To perform this alignment, a hydrophone was placed in the laser sheet, which was parallel to the beam axis [z axis in Fig. 2(a)]. The transducer was then moved using a positioning system, so that the beam axis traversed a path perpendicular to the sheet [x axis in Fig. 2(a)]. When the hydrophone registered a maximum value, it was assumed that the beam axis resided in the plane of the laser sheet. As an alternative to hydrophone measurements, the transducer could be moved in the x direction until the maximum streaming velocity was observed.

After the beam axis was aligned with the laser sheet, the ultrasound was turned off and the fluid motion was allowed

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**TABLE I. Physical characteristics of HIFU transducers used in the experiments.**

<table>
<thead>
<tr>
<th>Parameters</th>
<th>HIFU-1</th>
<th>HIFU-2</th>
<th>HIFU-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transducer radius</td>
<td>5 cm</td>
<td>3.8</td>
<td>3.2</td>
</tr>
<tr>
<td>Operating frequency</td>
<td>1.5 MHz</td>
<td>1.107 MHz</td>
<td>1.1 MHz</td>
</tr>
<tr>
<td>Focal distance</td>
<td>15 cm</td>
<td>11 cm</td>
<td>6.264 cm</td>
</tr>
</tbody>
</table>

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FIG. 1. (Color online) Flow chart summarizing the inverse methodology used to determine acoustic power and intensity.
to dissipate. The ultrasound was turned on and the flow field near the focus was allowed to reach steady state. Our numerical calculations predicted that flow near the focus reaches steady state in $\sim 3 \text{ s}$, while outside the focus it takes longer depending on the boundary conditions. Thus, after 6.6 s the CCD camera was triggered to capture images of the flow field. Our PIV data also showed that velocity fields captured after 5, 10, and 15 s were the same.

Though a single image pair can give the instantaneous flow field, in order to avoid random errors, 50 image pairs were captured in a 5 s duration. For the 50 image pairs, the relative standard deviation in the velocity was less than 2%.

All the image pairs were processed using a standard cross-correlation algorithm to get the streaming velocity field. Both the magnitude and direction of the velocity are provided by the algorithm. Additionally, no out-of-sheet components need to be resolved, since the flow is axisymmetric (no flow through the laser sheet) for axisymmetric transducers.

### B. Numerical computation of streaming velocity

The first step in calculating the streaming velocity was to compute the momentum transferred to the test fluid by the acoustic field. Predictions of the acoustic pressure were made by solving the Khokhlov–Zabolotskaya–Kuznetsov (KZK) nonlinear parabolic wave equation. The KZK equation for an axisymmetric sound beam propagating in the $z$ direction is (Hamilton and Morfey, 1998)

$$\frac{\partial}{\partial t'} \left[ \frac{\partial p}{\partial z} + \frac{D}{\rho_0 c_0^2} \frac{\partial^2 p}{\partial r^2} + \frac{\beta}{\rho_0 c_0} \frac{\partial p^2}{\partial t'} \right] = \frac{c_0}{2} \left[ \frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} \right]. \quad (1)$$

Here $p$ is the acoustic pressure, $t' = \frac{r^2}{c_0}$ is the retarded time, $t$ is the time, $c_0$ is the speed of sound in the medium, $r = \sqrt{x^2 + y^2}$ is the radial distance from the axis of the beam, $\rho_0$ is the ambient density of medium, $D$ is the sound diffusivity of fluid medium, and $\beta$ is the coefficient of nonlinearity defined by $\beta = 1 + B/2A$, where $B/A$ is the nonlinearity parameter for the fluid medium. As indicated by the second term-
poral derivative in the time-domain representation, the model employs the classic thermoviscous model of absorption, proportional to the square of the frequency in a frequency-domain representation (Hamilton and Morfey, 1998). The numerical solution is implemented using the time-domain code KZKTEXAS2 developed by Lee (1993). In the execution of the code the pressure was assumed to be constant across the transducer face, and the transducer characteristics were taken from Table I.

A steady streaming motion is generated by the absorbed acoustic energy through the spatial variation in the Reynolds stress associated with the acoustic field (Lighthill, 1978a, b). In terms of the acoustic particle velocity \( u_r \), the Reynolds stress is given by \( \bar{\rho} u_r u_j \), where the overbar denotes a time average. The force (per unit volume) associated with the Reynolds stress (Lighthill, 1978a, b) is

\[
F_j = -\frac{\partial (\bar{\rho} u_i u_j)}{\partial x_i}.
\]  

Here repeated indices denote summation. The streaming motion satisfies the equation of motion expressed as (Lighthill, 1978a, b),

\[
\rho_0 \left( -\frac{\partial \bar{u}_i}{\partial x_i} \right) = F_j - \frac{\partial \bar{p}}{\partial x_j} + \mu \nabla^2 \bar{u}_j
\]  

along with the conservation of mass equation

\[
\rho \frac{\partial \bar{u}_i}{\partial t} = 0
\]  

for an incompressible fluid.

Of primary interest in transducer characterization is the acoustic intensity in the focal region. In the focal region, the acoustic field may be modeled as a beam of locally planar waves traveling in the \( z \) direction. For this quasiplanar field, the force associated with the Reynolds stress [Eq. (2)] is in the axial direction and has the form (Nyborg, 1965)

\[
F_z = \frac{2\alpha}{(\rho_0 c_0)^2} \rho \bar{p} = \frac{2\alpha}{\rho_0 c_0} I,
\]  

where \( \alpha \) is the absorption coefficient of the medium, and

\[
I = \frac{\bar{p}^2}{(\rho_0 c_0)^2}
\]  

is the time averaged acoustic intensity which is calculated by solving Eq. (1). In principle, a more complex intensity relation than Eq. (5b) that accounts for the motion of the medium must be used (Morse and Ingard, 1968) in intensity calculations for a streaming medium. However, the error incurred in using Eq. (5b) is on the order of the Mach number \( u_0/c_0, \) \( u_0 \) being a measure of the streaming speed. The Mach number for the streaming jets reported on in this paper is on the order of 0.0001 (Figs. 5 and 8), and hence the use Eq. (5b) for a stationary medium expression is justified.

For an axisymmetric (e.g. spherically concave) transducer in an infinite medium, the governing equations for the streaming field [Eqs. (3) and (4)] may be rewritten as

\[
\frac{\partial \bar{u}_i}{\partial z} + \frac{1}{r} \frac{\partial (r \bar{u}_i)}{\partial r} = 0,
\]  

\[
-\frac{\partial \bar{u}_z}{\partial r} + u_z \frac{\partial \bar{u}_z}{\partial z} = 1 \frac{\partial \bar{P}}{\partial \rho} + v \left[ \frac{\partial^2 \bar{u}_z}{\partial z^2} + 1 \left( \frac{1}{r} \frac{\partial \bar{P}}{\partial \rho} + 1 \left( \frac{\partial \bar{u}_z}{\partial r} \right) \right) \right],
\]  

\[
\frac{\partial \bar{u}_r}{\partial z} + u_z \frac{\partial \bar{u}_r}{\partial z} = -1 \frac{\partial \bar{P}}{\partial \rho} + v \left[ \frac{\partial^2 \bar{u}_r}{\partial z^2} + 1 \left( \frac{\partial \bar{u}_r}{\partial r} \right) \right].
\]  

Equations (6)–(8) were solved using the Galerkin finite element method (Fluent Inc., 2002), in a geometry that simulated the tank of Fig. 2(a).

C. Iterative characterization

To illustrate the iterative approach, we take the unknown parameter of interest to be the acoustic power. For the first iteration, a guess is made for the power, enabling the KZK equation to be solved for the first iterate of the acoustic field. From the KZK equation, the axial component of the driving force \( F_z \) is calculated from Eq. (5). Equations (6)–(8) are then solved to obtain the first iterate of the velocity field.

Streaming velocity fields obtained from both experiment and computation are then input to a Nelder Mead multidimension optimization algorithm (Lagarias et al., 1995; MathWorks, 2002). The objective function in this algorithm is

\[
\text{Error}_{\text{rms}} = \sqrt{\sum_{i=1}^{n} (u_{i,\text{exp}} - u_{i,\text{num}})^2},
\]

where \( n \) is the number of velocity nodes in the camera’s field of view. Error_{\text{rms}} measures the deviation of numerical velocity profile from the experimental values. The optimization routine then attempts to reduce the error by adjusting the power estimate and recalculating the intensity field. The entire procedure is repeated until the rms error is minimized. (Further iterations produce no reduction in error.) Provided the numerical velocity field approximates the experimental one reasonably well, the acoustic power is taken to be the last iterate of the power, and the intensity field given by the most recent output of the KZK equation. The total number of iterations required for convergence of the optimization algorithm is typically 30–40, and the duration for the entire back-calculation procedure is of the order of 3 h. Computations were conducted on a 3 GHz processor with 2.0 Gbytes RAM.

D. Validation experiments

The total acoustic power and intensity field estimated from the inverse approach were compared with the measurements from radiation force balance (RFB) and hydrophone-scan techniques. The RFB used in this study consists of a highly absorbing target suspended in a water bath (Maruvada et al., 2007). The transducer to be calibrated was mounted directly above the target. When the ultrasound is turned on, the target experiences a net force arising due to the momentum associated with the ultrasound wave. The target was connected to a sensitive balance which measures the
force acting on it. From the measured force, acoustic power was obtained by multiplying it with the sound speed, and the factor $1 + (a/2d)^2$ ($a =$ transducer radius, $d =$ focal length) to account for focusing effects (IEC 2005). Powers determined from the RFB were estimated to have an uncertainty of less than 10%.

Pressure measurements were made using 0.6 mm piezoelectric ceramic hydrophones (Dapco Industries, Ridgefield, CT). The hydrophones were calibrated using a planar scanning technique (Herman and Harris, 1982). The uncertainty in the pressure measurements acquired using these hydrophones was approximately 20% (Herman and Harris, 1982). The transducer to be characterized was mounted horizontally in a Plexiglas tank surrounded with sound absorbing materials, and the hydrophone was moved within the HIFU beam. The movement of the hydrophone in all three coordinates was monitored and controlled using computer controlled stepper motors. A scanning step size of 0.4 and 1 mm was used along radial and axial directions, respectively. At each step, measurements of the peak positive pressure were made. Since the hydrophone scanning method is accurate only at low power levels, this validation was done at acoustic powers less than 5 W.

III. RESULTS

In Sec. III A, computed streaming velocities are compared with previously published results. Computed and measured streaming fields are also compared using a known sound source. Additionally, the stability of the streaming jet is analyzed. In Sec. III B our inverse algorithm is used to determine acoustic power and intensity in water, at low power. The inverse method is then used to determine the absorption of a more viscous medium, at moderate power. Characterization of a transducer of unknown power is then performed in the higher-viscosity medium.

A. Forward problem: Experimental and numerical streaming velocity fields

Figure 3 shows a single raw photographic image obtained from DPIV measurements and the corresponding streaming velocity contour obtained after postprocessing using a standard cross-correlation algorithm (IDT Provision, Tallahassee, FL). This velocity contour is the average of 50 image pairs captured in a 5 s duration. The standard deviation for 50 images at the point of maximum velocity is $\approx 1.93\%$.

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1. Validation of numerical model

Our computational model for calculating streaming velocity was validated with the data of Kamakura et al. (1995). Figure 4 shows the transient streaming velocity generated by a Gaussian shaded transducer with diameter 1 cm, focal

![Graph](image)
length 5 cm, and central frequency 5 MHz, driven at a maximum source pressure of 30 kPa. The transient speed calculated using the present model matches with the result published by Kamakura et al. (Fig. 2 of Kamakura et al., 1995) to within about 1%.

2. Comparison of experimental and numerical streaming fields

Figure 5(a) compares the radial speed profiles obtained experimentally using DPIV, and computationally by solving Eqs. (6)–(8) [highest and lowest curves in Fig. 5(a), middle curve containing “averaged data” will be discussed subsequently]. The sound source used for this validation experiment is the HIFU-2 transducer (Table I), driven at a known acoustic power of 5.4 W, as measured using the RFB. The peak velocity profile calculated from the computational model is ~15% higher than the peak velocity measured using the DPIV system. A similar trend is observed for other transducers and other power levels.

The reason for this offset involves the finite thickness of the laser sheet. As mentioned earlier, the thickness of the laser sheet [Fig. 2(a)] when it exits the Powell lens is around 0.5–1 mm. However, as the laser passes through the flow medium, the sheet diverges gradually and becomes ~2 mm thick when it reaches the ultrasound beam axis [Fig. 5(b)]. Therefore, instead of visualizing a single plane (2 × 2 cm), the CCD camera captures particle motion in a three-dimensional space of size 2 × 2 × 0.2 cm. As a result, the streaming velocity measured at a particular point in the YZ plane [Fig. 2(a)] is essentially the spatial average of the velocity occurring along the thickness of 0.2 cm of the sheet in the x direction.

In order to account for this measurement error in our numerical algorithm, the velocity field obtained numerically was averaged over the 0.2 cm thickness of the laser sheet in the x direction [Fig. 5(b)]:

$$\bar{u}_{\text{averaged}} = \frac{1}{L} \int_0^L \bar{u}(r,z) dx, \quad r = \sqrt{x^2 + y^2},$$  \hspace{1cm} (10)

where $\bar{u}$ is the velocity estimated from the computations and $L$ is the half thickness of the laser sheet (~1 mm). This averaged velocity, $\bar{u}_{\text{averaged}}$, was then used in the optimization routine to compute $\text{Error}_{\text{rms}}$ in Eq. (9). Figure 5 (middle curve) shows the velocity profile calculated from our computations after doing the spatial averaging. The averaged velocity profile more closely matches the experimental data; the difference is approximately 6%.

3. Higher-viscosity streaming medium

DPIV experiments performed in water at higher acoustic powers (5–30 W) revealed the onset of instability in the streaming jet. This instability was manifested in the form of small puffs occurring at irregular times, just below the focus. This behavior was observed for all three HIFU transducers. As a result of the behavior, unrepeatable velocity fields were obtained.

To overcome the stability problem, the viscosity of the medium was increased (Reynolds number decreased). An alternative fluid medium composed of degassed water mixed with Natrosol-L (Hydroxy ethyl cellulose, Hercules, Wilmington, DE) in various concentrations was developed, and used in experiments where the acoustic power exceeded 5 W. The mixture was degassed using a vacuum pump prior to sonication. The viscosities of 1.4% (1.4 g/100 ml water), 2.4% and 3.4% Natrosol-L solutions were measured using a Cannon type-E viscometer to be ~5, ~12, and ~25 cp, respectively. Speed of sound for the 2.4% Natrosol solution was measured using an ultrasonic time delayed spectrometry system (Harris et al., 2004) to be 1480 m/s (same as for degassed water). Finally, as discussed in Sec. III B, the attenuation coefficient of the 2.4% Natrosol solution was found using our streaming method to be roughly twice that of water.

Figure 6 shows radial speed profiles obtained using water as well as 1.4%, 2.4%, and 3.4% Natrosol solutions as the flow medium. The transducer used in this comparison study is HIFU-1 excited at an output power of 30 W. Broader, smoother, and more axisymmetric profiles can be seen with increasing Natrosol content. As the fluid viscosity increases from 1 (water) to 25 cp, the peak velocity reduces from 8.2 to 5.8 cm/s.

Figure 7 shows the fluid speed obtained in a 2.4% Natrosol solution for three different transducers driven at same voltage (42 $V_{\text{rms}}$). The plot shows that the velocity distribution at the focal region depends upon the focusing characteristics of the transducer. Focusing gain, 3, and 6 dB beam...
dimensions (measured from hydrophone scans) of the transducers are listed in Table II. The focusing gain (Lee, 1993), given by the expression $g = \frac{a^2}{2c_0 d}$ ($\omega$ being the central frequency of the transducer, $a$ the transducer radius, $c_0$ the speed of sound in the medium, and $d$ the focal length), is the highest for HIFU-1. The corresponding peak streaming velocity is maximum for this transducer. The velocity contours for HIFU-1 are relatively narrow, in accordance with the small 6 dB beam width (Table II). In contrast, the velocity contours for HIFU-3 are short in axial direction, consistent with the smaller 6 dB width in the axial direction (Table II). From the 6 dB dimensions shown in Table II, it can be seen that focal region of HIFU-2 is larger than HIFU-1 and HIFU-3. Consequently, velocity contours obtained using this transducer are elongated.

Figure 8 compares the streaming velocity profile obtained numerically and experimentally while driving HIFU-3 at an input voltage of 35 Vrms, in a 12 cp medium. The corresponding acoustic power measured using the RFB is 19 W. It can be seen that experimental and numerical velocity profiles match closely ($\sim 1\%$) in both the axial and radial directions.

The width of the velocity distribution in Fig. 8(b), based upon the distances at which the speed has dropped to half its maximum value (kinetic energy reduced to $\frac{1}{4}$ the maximum), is around 0.5 cm. By contrast, the intensity width for the same transducer (HIFU-3) is about 0.2 cm (Table II). In general, the width of the velocity distribution is larger than that of the intensity distribution, and the effect becomes more pronounced as the viscosity of the medium increases. In Fig. 6, for example, the width of the velocity profile for HIFU-1 in the 2.4% Natrosol solution (viscosity 12 cp) is about 5.4 mm. The width of the velocity profile in water (viscosity 1 cp) is between 2.4 and 3.3 mm, depending upon how the asymmetry is treated. Within the limits of linear acoustics, the width of the intensity distribution is independent of viscosity, with the value of about 1.7 mm for HIFU-1.

**B. Inverse problem: Total acoustic power and HIFU beam characteristics**

In the following, the optimization algorithm is used to backcalculate various parameters characterizing the transducer and the propagation medium.
1. Characterizing acoustic field at low power levels (<5 W)

Since current field hydrophone mapping techniques are practical only at low power levels, characterization was first performed at low input voltages with water as the fluid medium. Figure 9 compares the source pressure performed at low input voltages with water as the fluid medium. Figure 10 compares the source pressure of HIFU-2, obtained via hydrophone scanning and the inverse method for a transducer voltage of 10 \( V_{\text{rms}} \). Peak intensity at the focus measured by the inverse method is 65 W/cm\(^2\), approximately 4% below the peak intensity measured by hydrophone (67.8 W/cm\(^2\)). The 6 dB length and width, backcalculated from the streaming velocities, are 2.7 cm \( \times \) 2.7 mm; the corresponding values obtained from hydrophone scanning are 2.95 cm \( \times \) 3 mm.

2. Characterizing acoustic field at moderate power levels (<30 W)

As noted earlier, a Natrosol-based solution is the preferred medium for characterization when the acoustic power exceeds about 5 W. However, unlike water, little is known regarding the absorption properties of Natrosol solutions. Equation (5) shows that the Reynolds stress, which accounts for momentum transfer from the sound field to fluid, is proportional to the absorption coefficient of the medium. Using a transducer of known intensity, this parameter can be treated as the unknown in the optimization algorithm, in the following manner.

Figure 11(a) summarizes the procedure for calculating the acoustic absorption coefficient of the medium. Using DPIV, the streaming velocity generated by the HIFU-2 transducer in the 2.4% Natrosol-L solution was measured. First, an initial guess for the unknown \( \alpha \) was made. In these determinations of absorption coefficient using the inverse algorithm, the attenuation coefficient of water (2.52 \( \times \) 10\(^{-3}\) Np cm\(^{-1}\) MHz\(^{-2}\)) was taken as the first guess for \( \alpha \). Then, the forcing term was calculated using Eq. (5) and the streaming velocity was calculated numerically by solving Eqs. (6)–(8). The \( \text{Error}_{\text{rms}} \) value was then computed from the experiment and numerical velocity profiles and input into the optimization routine. The procedure was repeated until the relative error was reduced to a value of 10\(^{-3}\).

The absorption coefficient of 2.4% Natrosol measured using this procedure is shown in Fig. 11(b). The \( \alpha \) values measured at the power levels 3.6, 5.8, and 11.8 W were fairly constant, approximately 5.65 \( \times \) 10\(^{-4}\) Np/cm for the 1.1 MHz frequency considered. This value of \( \alpha \) is used in the subsequent calculations to characterize transducer output at moderate and high power/Intensity levels.

Figure 12 compares the power measurements made using the RFB and streaming in the 2.4% Natrosol medium. Transducers HIFU-1 and HIFU-3 were driven at voltages ranging from 19 \( V_{\text{rms}} \) to 42 \( V_{\text{rms}} \). The inverse method was able to measure the acoustic power with a maximum error of about 10%. Computed intensity profiles in the Natrosol medium are shown in Fig. 13, for a power of 30 W. The 6 dB length and width, backcalculated from the streaming veloc-

<table>
<thead>
<tr>
<th>Transducer</th>
<th>6 dB dimension (axial ( \times ) radial)</th>
<th>Focusing gain</th>
</tr>
</thead>
<tbody>
<tr>
<td>HIFU-1</td>
<td>2.4 cm ( \times ) 1.7 mm</td>
<td>53</td>
</tr>
<tr>
<td>HIFU-2</td>
<td>2.95 cm ( \times ) 3 mm</td>
<td>31</td>
</tr>
<tr>
<td>HIFU-3</td>
<td>1.35 cm ( \times ) 2 mm</td>
<td>38</td>
</tr>
</tbody>
</table>

FIG. 8. (Color online) Velocity profiles obtained from computations and experiments in 2.4% Natrosol solution; (a) velocity profile along axial direction and (b) velocity profile along radial direction. Transducer is HIFU-3; input voltage=35 \( V_{\text{rms}} \) (19 W). For the experimental data, standard deviation error bars for five trials are also shown. The maximum standard deviation for five trials is 0.02 cm/s.
ties, are 1.2 cm $\times$ 1.85 mm. The values determined using hydrophone scanning at low power, both in water and Natrosol, are 1.35 cm $\times$ 2 mm (Table II).

IV. DISCUSSION

The streaming technique as presented in this paper is useful for characterizing HIFU transducers in the “moderate” intensity regime, roughly 100 to 1000 W/cm$^2$. The intensities are low enough that linear acoustic propagation is a reasonable assumption, yet high enough to possibly damage conventional hydrophones.

At low powers—around 3 W or 70 W/cm$^2$ focal intensity, close agreement with piezoceramic hydrophones (Fig. 10) was obtained. At focal intensities above about 1000 W/cm$^2$ (transducer powers around 20 W), divergence between the measured and predicted power levels can be seen (Fig. 12), probably due to nonlinear propagation effects. While the KZK equation can predict the generation of additional frequency components by nonlinear mechanisms, the absorption model used in our iterative method does not account for the frequency dependence of the absorption. A more comprehensive model of the absorbed acoustic energy would involve a sum of the form $\sum \alpha_n I_n$, where $I_n$ is the intensity of the $n$th acoustic harmonic and $\alpha_n$ the corresponding absorption. The iterative scheme would then involve optimization over an $n$-dimensional space containing the intensity modes. These nonlinear-propagation effects will be treated in a future generation of the model.

The inverse method was first used to determine unknown transducer power. Because we employed the linearized version of the KZK equation, the intensity field is linearly proportional to the transducer power, and the KZK equation actually needed to be solved only once. Subsequent intensity fields required in the iteration process (iteration is still required due to the more complex dependence of velocity upon intensity in the Navier-Stokes equations) could be obtained by simply multiplying the first intensity field by a scaling factor dependent upon the current value of the acoustic power. This would not be true for other transducer-characterization parameters, such as focal length, beam tilt angle relative to the desired axis, or the absorption of the medium.

Determining an unknown absorption (Fig. 11) is a characterization problem for which the streaming technique is well suited. Conventional absorption-measurement techniques such as time delay spectroscopy (Peters and Petit, 2003) measure attenuation over a prescribed propagation distance. When the attenuation is small over the given distance, these techniques are susceptible to large error. With the streaming-based technique, however, even slight attenuations such as that of water can yield readily measurable fluid velocities. Mathematically, the forcing term in the momentum equation [Eq. (5)] contains a factor of $\alpha$, producing relative changes in the fluid velocity field that are commensurate with relative changes in the absorption coefficient. This commensurate change in velocity with variation in $\alpha$ also allows for stable operation of the optimization algorithm; nonuniqueness issues arise when the streaming field is insensitive to changes in the parameters being determined.

The optimization technique may be used to estimate multiple parameters simultaneously, though the computation time increases significantly, and convergence can be more difficult to achieve. For example, determining the transducer intensity simultaneously with the absorption coefficient would be difficult, since the two quantities $\alpha$ and $I_0$ appear primarily as a product [Eq. (5)]. The absorption coefficient also appears in the exponential attenuating factor of the intensity but, as is the case with conventional absorption measurement techniques, this exponential factor is relatively insensitive to changes in $\alpha$.

The streaming algorithm in this paper assumes an axisymmetric ultrasound beam. The technique can be extended to three-dimensional $(x,y,z)$, nonsymmetric beams with no change in the procedures performed (Fig. 1) in the algorithm. A three-dimensional propagation code and fluid-flow simulator would be necessary, and an increase in computation time would result. The three-dimensional version of the technique would be useful in diagnosing imperfections or problems in transducer fabrication, such as a misfiring array element or a propagation axis that was skewed relative to the axis of sym-
metry of the transducer face. The problem would first be manifested in an asymmetry of the experimental streaming velocity field. The three-dimensional inverse algorithm could be used to help predict the transducer characteristics giving rise to the asymmetric velocity field that was observed. For example, the location of a nonfunctioning array element could be left as a free parameter in the optimization procedure, to be determined as part of the iterative solution.

Figure 6 illustrates the complex dependence of velocity upon fluid viscosity in a streaming medium. An increase in viscosity increases acoustic absorption, i.e., produces a larger transfer of momentum from the acoustic field to the hydrodynamic field, resulting in a higher peak (on-axis) velocity. However, the higher viscosity also serves to radially diffuse momentum, resulting in a lower peak velocity. Thus, the peak velocity in Fig. 6 is essentially unchanged as the viscosity is increased by a factor of 5 from 1 to 5 cp, but drops about 25% when the viscosity is doubled from 12 to 25 cp.

The higher viscosity test fluids are essential at higher streaming velocities (higher intensities) due to stability requirements for the streaming jet. The higher viscosity test fluids also yield more symmetric velocity profiles, as can be seen by comparing the natrosol-based curves in Fig. 6 with that for water. An upper limit on the viscosity of the test fluid is imposed by the low resulting velocities resulting at high viscosities. These velocities can become difficult to quantify accurately using DPIV systems. For the intensity range considered in our studies, the agreement between experimental and computational velocity profiles was best for the 12 cp fluid (Fig. 8).

The KZK equation was used in our simulations, however, any beam propagation code can be used in the iterative algorithm outlined in Fig. 1. As noted by Tjotta and Tjotta (1993), the accuracy of the KZK degrades when the beam becomes highly focused (large aperture transducers). Tjotta and Tjotta (1993) noted that the ratio $a/d$ of transducer ra-

![FIG. 10. (Color online) Acoustic intensity profiles as a function of (a) axial and (b) radial distances (cm), obtained from backcalculation and hydrophone scanning in water for HIFU-2 transducer. Transducer input voltage $= 10 V_{rms}$ (3.2 W).](image)
radius to focal length should be small in order to ensure accuracy of the KZK approach. For our transducers HIFU-1, HIFU-2, and HIFU-3, the \(a/d\) values were 0.33, 0.35, and 0.51, respectively. While the \(a/d\) value for HIFU-3 was relatively large, the agreement with experimental measurements of streaming velocity (Fig. 8) and total power (Fig. 12) indi-
cates that the criterion of Tjotta may be conservative. In any event, when high accuracy is required, a full-wave model such as the Helmholtz equation in the linear propagation regime or the Westervelt equation in the nonlinear regime should be employed in the iterative algorithm. The tradeoff is the increase in computation time associated with using a full-wave model compared to a parabolic equation.

A limitation of our inverse methodology is that the repetitive solution of the acoustic propagation equation and fluid-flow equations can involve significant computation time (~3 h). Our technique is also limited by the accuracy of the sound propagation code. For example, the KZK code currently employed by our model degrades in accuracy when the transducer gain becomes large. The above-mentioned drawbacks can be circumvented by developing an alternate method which determines the acoustic field directly from the experimental streaming velocity. In subsequent studies, an attempt will be made to estimate the acoustic field directly from the experimental streaming velocity data.

V. CONCLUSION

The streaming technique presented in this paper allows free-field characterization of HIFU transducers to be performed in an intensity range that may be hazardous to conventional hydrophones. The method is noninvasive and agrees well with hydrophone and radiation-force balance techniques in common ranges of applicability. In addition to predicting acoustic intensity fields, the approach may also be used to determine other important HIFU parameters, such as medium absorption, transducer focal length, or beam tilt angle.

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